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INSIGHTS FROM MATHEMATICAL MODELING IN SOVIET MISSION ANALYSIS (Part I)

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PROJECT OVERVIEW

The objective of this project is to examine and assess the extent to which U.S. military policy has effectively interpreted and responded to the military implications of Soviet weapons innovations. The project focuses on the contributions of Soviet weapons innovations to military mission performance, not changes in the technological level of Soviet weaponry. It also examines the ability of the Soviet weapons innovations process to offer a militarily significant breakout option.

Accordingly, three related lines of inquiry are being pursued. First, we are examining the Soviet approach to weapons innovation as it is portrayed in their force planning and weapons evaluation literature. This initial work enables us to better understand the preferences, assumptions, and biases that influence the armaments selection process (and hence the weapons innovation process) in the Soviet Union.

Second, we are analyzing the Soviet approach to measuring the relative contributions of weapons innovation efforts towards improving mission capabilities, and not the extent to which a given piece of hardware can outperform the previous technological generation. Assessing mission contributions involves comparing quantities of arms and interaction with other weapons systems assigned to the given mission, as well as the qualitative characteristics of new weaponry.

Third, we are assessing the degree to which the Soviets have the capacity for "breakout"--significantly improving their military capabilities in a short period of time--through weapons innovation. The threat of a Soviet "technological surprise," in particular, has been a constant U.S. fear.

Part I of this report addresses one aspect of the second task described above. How do the Soviets use mathematical models to measure changes in mission capabilities and the impact of weapons innovations? Specifically, we examine the role that mathematical modeling plays in Soviet military analysis and some of its applications in the area of air defense analysis.

1.0 INTRODUCTION

In the first report we examined aspects of the Soviet weapons selection process and the ways in which that process influenced (and was influenced by) weapons innovation. It was found that mathematical modeling played a significant role in the selection process involving modern high-technology weaponry. This finding suggests a new avenue for investigating Soviet military thinking about weapons innovation. Can we better understand the Soviet approach to force development by examining the mathematical structures they employ in the analysis of primary missions? What variables do Soviet military specialists emphasize, and what do they consider as "constants?" Can one detect one or more underlying conceptual themes in their approach to mission analysis?

This report begins by taking a broader look at the Soviet application of mathematics in military affairs, including some of the "philosophical" debates surrounding the initial introduction of modeling into Soviet military analysis.[1]

1.1 The Revolution in Military Affairs

The Soviet military marks the beginning of the contemporary revolution in military affairs with the development of nuclear weapons and missiles (in particular, missiles with intercontinental ranges). This revolution is not simply one of technology, but encompasses military science (how to think about and analyze military affairs) and military art (how to plan and to execute military operations).

Soviet military science has always claimed to be highly empirical, drawing heavily on historical experience. In fact, the Soviet General

Staff has an independent directorate whose purpose is the detailed study and analysis of the historical experience of combat and war. (Its work is published in the Voyenno-Istoricheski Zhurnal [Military Historical Journal].) The revolution in military affairs ushered in a class of weaponry for which there was little relevant historical experience. While historical analogies were recognized as valuable to the study of modern war, they were not sufficient. Modeling, simulation, and gaming became important tools in the study of military science and military art. Indeed, Soviet specialists draw a parallel between the study of military art and that of the natural sciences where mathematical models are used extensively.

> In spite of the many peculiarities of combat operations, they lend themselves excellently to the use of mathematics in military analysis. Military art is not replaced by these analyses, but instead receives a powerful supplementary tool which has justified itself many times in various realms of science.[1]

In the view of Soviet military specialists, the theory of operations research--the application of quantitative methods to the study of man's organized activities--is not sufficiently developed to offer definitive solutions to military planning problems. Yet, it can greatly assist military decision making by providing quantitative recommendations that commanding officers can either accept or reject based on an analysis of other factors not included in the mathematical models. Quantitative analysis can provide a substantiated "objective" and "scientific" basis for decisionmaking.

1.2 Sources and Methods

Our interest is in Soviet military programming in the 1970s and the

1980s. As was noted in our first report, the nature of the process of Soviet weapons selection and force posture development is such that there is an 8 to 12 year lag between initial conceptualization and operational appearance of major weapons systems. Thus, the period of the mid-1960s to the mid-1970s would appear to be the most fertile ground for investigating Soviet military thinking on the application of mathematics relevant to military planning for the 1970s and 1980s.

The richest and most valuable source of data is the journal, Military Thought (Voyennaya Mysl'). It is the restricted journal of the Soviet General Staff, and is not intended for foreign audiences. CIA translations are publicly available for the years 1963-1973. A total of 25 articles, letters to the editors, and book reviews published in Military Thought were reviewed. The distributions of these articles by year of publication and rank of author are shown in Table 1. Two aspects are worth noting. First, a flurry of interest in mathematical analysis appears to develop in the early 1970s. Second, specific mathematical models (rather than heuristics) begin to appear in later articles, suggesting a basic acceptance of mathematical modeling in military planning by the early 1970s. While these articles were written by high ranking officers, these ranks are somewhat lower than appears typical of Military Thought authors (see Table 2). This may well be a function of generational change and training (General-Lieutenants would be too old to have the proper educational background), and not an indicator of the relative importance of the material.

Since several of the articles discussed in detail the general problem of applying the methods of mathematics and operations research to the study of military affairs, this will be discussed first. Next, several applications of mathematical models will be described including the use of game theory in decisionmaking, method for determining the combat readiness of military equipment, and techniques for calculating the effectiveness of air defense systems. The final section will examine indications of the use of mathematical methods by the Soviet military.

1.3 Findings

Mathematical modeling first enters the domain of Soviet military science in the mid-1960s. This surge of interest in mathematics in military affairs is marked by a lengthy series of articles in the restricted Soviet Journal Voyennaya Mysl' and by the near simultaneous emergence of several volumes published by the Ministry of Defense. It is worth noting that the key articles and books were, for the most part, written by (or published under the signatures of) general-grade officers.

Soviet interest in mathematical modeling was stimulated by the demands of programming, planning, and budgeting (resource allocation) arising out of the revolution in military affairs. Complementary work on systems analysis being conducted by the U.S. Defense Department was of particular interest to Soviet military specialists. Efforts in mathematical modeling were concentrated among three specific areas: weapons evaluation and selection, combat analysis, and troop control (specifically related to automated C^3 systems).

While civilian theoretical work on mathematics often is incorporated in military discussions, Soviet specialists in the application of mathematics to military affairs are all professional military officers. In fact, Soviet authors deride the Western norm of civilian operations analysts carrying out military studies. The Soviet view is that one cannot conduct valid analyses without having first acquired an extensive background in military art (strategy, operational art, and tactics). Thus, professional military officers who are first schooled in military art and then trained in mathematics become Soviet military operations analysts.

Thus, Soviet modeling work is driven by the fundamental concepts of Soviet military science and military art as interpreted by the professional military. These are translated into formal axioms for mathematical expression.

A considerable amount of the Soviet effort in mathematical modeling is tied to interest in "automating" troop control, decisionmaking, and weapons system functions. Particular emphasis is given to: automated systems for information collection, transmission, sorting, and retrieval; automated decision aids for commanders (e.g., air defense units); and *automated weapons system* activity (cybernetics). This is one area in which the Soviets believe that a breakout-like capability could develop. Automation technology holds out the potential of vastly raising the combat effectiveness of existing and future weapons.

In the area of weapons system analysis special attention is given to evaluating combat readiness, which includes the human user. Combat readiness models are broken down into three components: the promptness of execution of combat missions (including C^3 functions), system reliability defined in terms of operational readiness under different scenarios and the probability of failure-free operation, and crew operation.

Soviet specialists strongly suggest that strategic mission analysis and related weapons selection should be conducted on the basis of a "maximin" approach. That is to say, one chooses the approach that corresponds to the simultaneous assumptions of worst possible risk and an outcome of minimal acceptable gain. While risk and daring may be acceptable in tactical decisionmaking, they are not acceptable in the strategic realm.

In the armaments selection process, a maximin approach is warranted when the adversary has a greater or equal scientific-technical and economic potential. It is also warranted when the adversary "knows" the general outline of one's plans, such as is almost always the case in long range strategic force development.

The treatment of uncertainty and risk is divided into two dimensions: that controlled by the enemy and that which is unknown (stochastic). In the former case the data distribution is considered undefined, while the latter case is dealt with by distributional approximations.

Mathematical modeling in air defense studies is heavily oriented towards automated troop control and weapons system control. In combat modeling the allocation of air defense resources is generally "a given" and instead combat dynamics are studies. This suggests a supply push air defense allocation approach in which higher levels allocate air defense resources and where lower level decisionmaking involves making the best use of assumed fixed assets.

Air defense analyses are structured as mass servicing problems. Primary interest appears to be in investigating the saturation limits of fixed asset air defense barriers and air defense system mixing (when the barrier is composed of heterogenous means and forces). "Time"--control time of command, control, and communications systems and performance time of weapons--consistently emerges as a variable to optimize. While "friendly assets spared" is often given as the preferred outcome measure, in practice Soviet mathematical models predict fraction of enemy forces destroyed as a surrogate measure. Changes in friendly and enemy capabilities emphasize macro capabilities that can be transformed easily into mass servicing format (i.e., service time, waiting time, etc.). Most often, weapons system evaluations are based on preset norms--e.g., a 40% attrition rate for intruders--and not on some maximizing algorithm. However, modern aviation technology--especially unpiloted vehicles--undermine the assumption upon which Soviet air defense planning has been built over the past two decades.

Our examination of Soviet mathematical modeling efforts in the air defense area suggests that they are most likely to look for a "breakout-like" impact to be achieved through automation of weapons and troop control.

Table 1. Distribution of Articles in Military Thought by Time of Publication

Year	Number of Articles	Number of Letters or Book Reviews
1963	1	2
1964	1	5**
1965	1	0
1966	2	0
1967	1	0
1968	0	4
1969	0	0
1970*	(n/a)	(n/a)
1971	1	0
1972	3	0
1973	5	1

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*No issues are publicly available in 1970.

**Includes two letters not available but whose existence was referred to elsewhere.

Table 2. Distribution of Articles in Military Thought by Rank of Authors

	In Sample		Average*
Rank	#	%	%
General Lieutenant or higher	0	0	18
General Major	10**	26	17
Colonel	13	34	51
Lieutenant Colonel	8	21	12
Major or Lower	7	19	2

----

*Average percentages are based on a random sample of the authors of articles in 20 issues of Military Thought, including a total of 207 authors.

**Includes four articles by I. Anureyev, two by N. Smirnov, and one each by K. Tarakanov, M. Botin, V. Rozhdestvenskiy, and A. Moskvin.

2.0 GENERAL THEORY OF MILITARY OPERATIONS RESEARCH

2.1 Definition of Operations Research

In a controversial article published in 1963, General-Major Anureyev (a senior Soviet specialist on mathematics) defined the theory of operations research as follows:

> The theory of operations research defines and analytically describes conformity to natural law in various processes with the goal of obtaining quantitative grounds or recommendations founded on their basis for the adoption of decisions.[2]

Operations research, or the "theory of decision-making," was differentiated from cybernetics, or the "science of guidance" (or control). While the two fields may rely on similar mathematical methods, Anureyev maintained that operations research was not a subset of cybernetics.[3]

This article and particularly these two points sparked a major debate in which four responses appeared along with a summary article which effectively closed the debate. Two of the responses by Gen. Maj. N. Smirnov and Prof. Ye. Venttsel' were referred to in the summary article but are not themselves available in the Military Thought collection. The content of these letters can be determined only by the excerpts cited in the final article. In one response, Maj. Pevnitskiy rejected Anureyev's definition and referred to the distinction beteen cybernetics and operations research as "clearly mishmash."[4] The Anureyev definition was seen as too broad by Pevnitskiy since it included nearly all fields of science. An an alternative, he maintained that one should first accurately define "operations" as "any action being organized"[5] and then define "operations research" as follows:

The theory of operations research is a scientific discipline which is concerned with analysis of similar elements of various operations (activities being organized), with consolidation of various elements into structures, with exposure of similar structures, and with quantitative analysis of them for the purpose of reaching a scientific basis for rational decisions acceptable by executing organs in control processes.[6]

In contrast to Anureyev, Pevnitskiy claimed that the two fields of cybernetics, which he defined as the "analysis of any process of control in nature and society," and operations research, which has the narrower focus of man's organized activities, cannot be separated.[7] In addition both Pevnitskiy and the authors of a second letter (Lt. Col. N. Bazanov* and Capt.V. Malinovskiy) rejected Anureyev's reference to operations research as a "theory of making decisions." Instead, operations research only provides the quantitative basis upon which decisions are made, but it is not a theory of how decisions are made.[8,9]

In the final article of the debate, Gen. Maj. A. Moskvin noted that while the Anureyev article was interesting and useful, it "contained many imprecise formulations and debatable statements which have been the cause of much enlivened discussion."[10] Moskvin noted that the main areas of debate were: (1) the definition of operations research, (2) the role of

*Bazanov wrote at least three critiques of Anureyev's works. In addition to the article discussed above, he also wrote one of the criticisms of Anureyev (1967) and a review of Application of Mathematical Methods in Military Science (a book Anureyev wrote with A. Tatarchenko).

the commanding officer, (3) the classification of the methods of analysis, and (4) the relationship between operations research and cybernetics. The second and third points will be discussed in subsequent sections.

Moskvin agreed with the criticisms mentioned above that operations research should not be defined as a "theory of decision making," but added that the laws defined by operations research need not be "analytical" as mentioned by Anureyev since they could also be statistical.[11] The alternative definition offered by Pevnitskiy suffered from two shortcomings: (1) some terms, such as "elements," were not adequately defined; and (2) some operations may not have similar structures or elements. Gen. Maj. Smirnov defined operations research in exclusively military terms: "the analysis of the many various processes and phenomena of armed combat; the character of the combat activity of troops; the problems of commanding them in a nuclear and rocket war; and the methods of troop control."[12] Moskvin maintained that this definition was unnecessarily restrictive since operations research can also be applied to non-military problems.

In providing an alternative definition, Moskvin first defined an "operation" in the broad sense as "the process of the work of people and machines organized for the execution of determined tasks," and then the purpose of operations research was "to work out quantitative bases to make decisions or give orders."[13] Acceptance of this definition of operations was resisted by the military who preferred to use "operations" in the narrow military sense, but Moskvin noted that this definition was too widely accepted both in the U.S.S.R. and abroad for a separate concept to be used in military applications. Definitions which appeared in subsequent Military Thought articles emphasized that operations research can provide only the quantitative basis upon which decisions can be made and not the decisions themselves.[14]

Moskvin dismissed the fourth area of the debate, i.e., the difference between operations research and cybernetics, as essentially irrelevant. In doing so he agreed with Venttsel' who suggested that nothing could be gained by this discussion.[15]

The time sequence of the articles in this debate is summarized in Table 3 below. The time interval between successive articles is relatively short compared to that for other "debates." For example the criticisms of Anureyev's nuclear "correlation of forces" article appeared approximately 14 months after the original article. The small intervals might tend to suggest that the debate was orchestrated by the editors of Military Thought. Three points, however, argue against this interpretation: (1) Moskvin claimed that the original article initiated an "enlivened discussion;" (2) this claim is reinforced by the tone of some of the responses, including Pevnitskiy's "mishmash" comment; and (3) Moskvin's dismissial of a significant portion of the debate as pointless (the editors would not manipulate a debate that they thought was pointless).

While the issues raised by Anureyev's critics are legitimate ones, they appear to be based more on semantics than on substance since Anureyev did refer to operations research as only providing the "quantitative numerical basis" for solving problems which is used in making final decisions along with other factors "which cannot be expressed quantitatively."[16] This issue may have been caused by concerns regarding the adoption of mathematical methods would have

Table 3. Time Sequence of Articles in 1963-1964 Operations Research Debate

Authors	Issue	Date signed to Press	Time Interval Between Articles
Anureyev, et al	July 1963	5 June 1963	----
Smirnov	Dec. 1963	(n/a)	5 mo.
Bazanov, Malinovskiy & Pevnitskiy	Feb. 1964	6 Feb. 1964	2 mo.
Venttsel'	Mar. 1964	(n/a)	1 mo.
Moskvin, et al	Sept. 1964	24 Aug. 1964	5 mo.

on the role of military commanders. Many officers may have reacted strongly to even faint suggestions that their autonomy would be jeopardized through the use of the methods of operations research. These criticisms did not disagree either in principle or in practice with the use of quantitative methods to solve military problems. In fact, a consensus appears to have developed that the introduction of nuclear weapons combined with the increased speed, scale and complexity of modern warfare makes the use of mathematical models in military science essential. Under these conditions, experiences in past wars and in present day local wars, while relevant, are not sufficient; knowledge about modern warfare can be obtained only indirectly through the use, for example of mathematical models.[17,18,19,20,21]

2.2 Isomorphism

The property of isomorphism, which is based on the "material unity of the world,"[22] makes possible the application of mathematical methods to the theory of operations, in general, and to combat operations, in particular. Isomorphism, or "similarity in form with significant differences in content," is the "theoretical prerequisite of the feasibility of modeling the processes of reality," including armed combat.[23] This properly applies to military affairs because:

> No one operation ... is a copy of another. But nevertheless, possibilities for the mathematical analysis of military operations do exist. In the apparent conglomeration of random factors, apparently completely unrelated to each other, there exist certain regularities, including mathematical ones.[24]

While similarities exist between some phenomena, making mathematical modeling feasible, at the same time no two phenomena are exactly identical.

Consequently knowledge obtained through the use of such analogies has a probabilistic character. This difficulty does not make modeling either useless or futile. The knowledge gained may be incomplete or one-sided, but gaining a complete understanding of any phenomenon in its totality all at once is impossible--approximations are always required.[25]

An example of isomorphism is the application of queuing theory to modeling air defense operations. Two apparently dissimilar activities as the servicing of automobiles at a gasoline station and the defense against a bomber attack by surface-to-air missiles can both be described by the same mathematics since both problems have certain key characteristics in common: the average arrival rate of "customers" (number of cars or bombers arriving per minute), the "servicing" of these "customers" by a fixed number of units (gasoline pumps or SAM complexes), the average "servicing time" for each "customer," and finally the possibility that "customers" may wait for "service" for only a fixed amount of time before leaving (due either to impatience or the limited range of SAMs).[26]

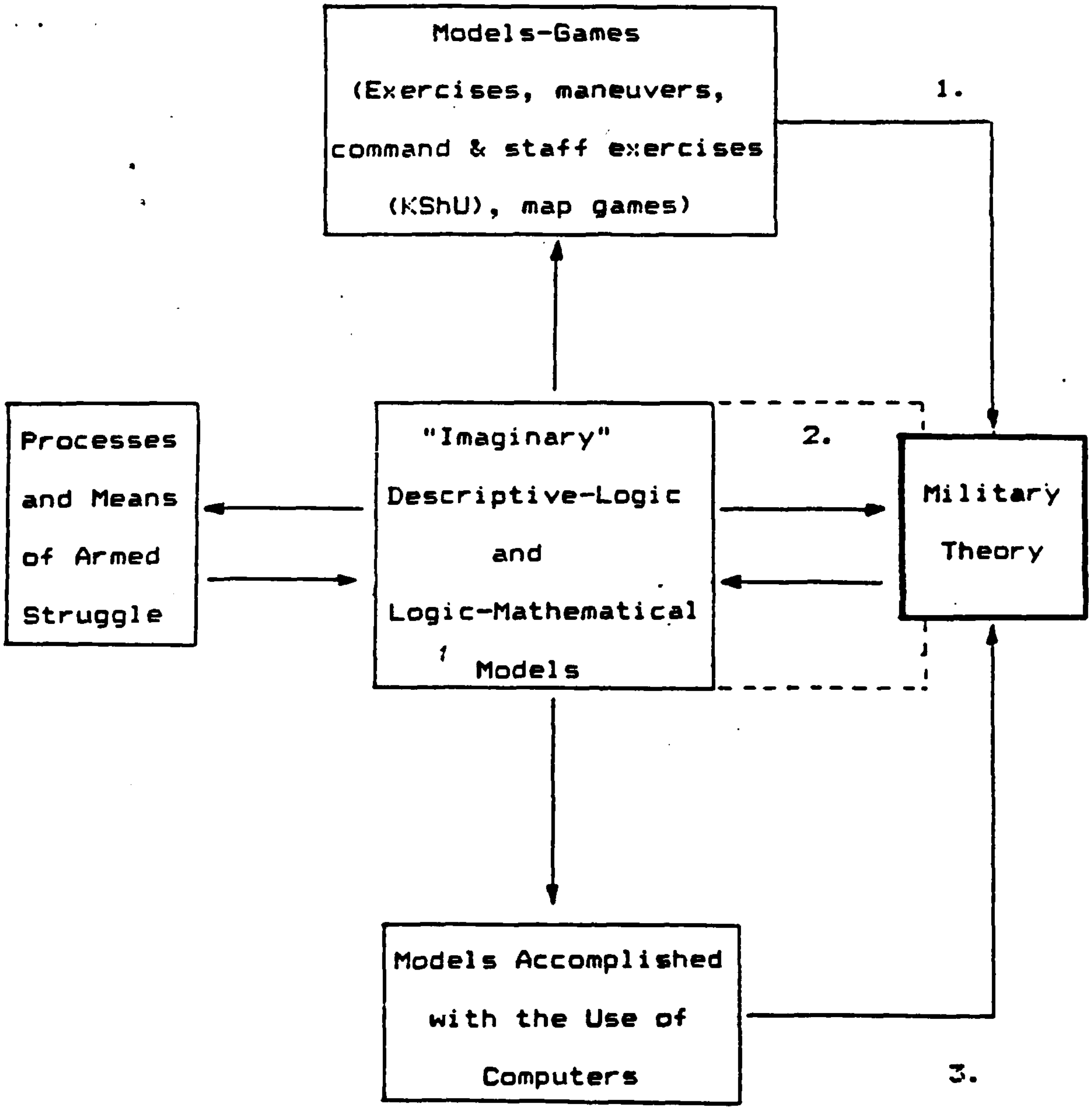
2.3 General Method for Developing Models

The first problem which must be confronted in developing models of combat operations is the translation of the theories of military science into mathematics which requires "unambiguous concepts connected by strict logical rules reflecting clearly fixed laws and associations of the process of warfare."[27] The difficulty arises because military science is based on natural language where concepts are vague and relationships ambiguous; thus total formalization may not be possible due to this incomplete knowledge. Gen. Maj. K.V. Tarakanov discusses this issue by distinguishing among "meaningful theory" based on natural language, "formal models" based on logic and mathematics, and the "original" (or the subject under study--actual combat operations). Mathematical models are developed on the basis of a meaningful theory and help to perfect that theory; the results of this theoretical understanding and mathematical computations can then be used to increase one's knowledge of the "original." Mathematical models have the advantages of simplicity and isomorphism and are useful in conducting experimental research, whereas meaningful theory "reflects the essence of the phenomena which represents a system of connections and relationships which are fundamentally unclear and always general."[28]

The process of developing mathematical models in order to advance military theory was described in similar terms by Maj. A. Dmitriyev and is summarized in Figure 1 on the next page. Dmitriyev distinguished between "real" or substantive models where the models themselves are based on physical processes (e.g., the use of scale model airplanes in wind tunnels) and "imaginary" models which are either verbal or mathematical analogies intended to represent armed combat. Dmitriyev claimed that there were three basic methods for conducting military research. The first method started with the construction of a "descriptive-logic" model which could be used to structure a gaming exercise. The results of this exercise would then be used to advance military theory. The second method was based on the use of a "descriptive-logic" model to create a "logic-mathematical" model, the results of which could lead directly to generalizations in military theory. The last method was similar to the second except that the mathematical model was sufficiently complex to require the use of a

Figure 1. Methods for Conducting Research in Military Theory

Source: Dmitriyev (1965; 8).



computer. Various combinations of these methods, such as a mixture of the first and third, could also be employed.[45]

The development of a mathematical model consists of five stages: (1) statement of the problem and the operational-tactical description of the model; (2) formalization of the mathematical model and development of the algorithm*; (3) validation of the correspondence between the model and the actual process under investigation; (4) complete documentation of the model and instructions for its use; and (5) introduction of the model into use.[30] The first, third, and fifth stages are the more important ones--and the ones which receive the most attention in Military Thought. The second stage is also extremely important but is discussed primarily in textbooks; articles in Military Thought only mention the fields of mathematics involved and the importance of advances in mathematics and computer science.

The first stage is the "most important and crucial" because it is where "mathematics and military sciences are united."[31] In this stage the concepts and theories of military science must be systematized and translated into a set of formal axioms, which can then be expressed mathematically. Tarakanov referred to this process as the development of a formal theory, a process which is useful in itself because it allows one "to look at the subject of investigation from an unexpected side and obtain new results which would be impossible within the framework of a meaningful theory (based on natural language)."[32] This formal theory

*According to Col. L. Kolosov (1973; 137) the word algorithm originated in the 9th century from the Latin "algoritimi" which is a transliteration of the name of a 9th century Uzbek mathematician, Al' Khorezmi.

does not include all of the features of the subject under study; it incorporates "only the most important ones which can be expressed quantitatively."[33] Therefore both correctly defining the problem and the mission to be accomplished and identifying the most important variables and their relationships are essential. Moskvin, in summarizing the 1963-1964 debate, agreed with Venttsel' that the outcome of a process is dictated by three groups of parameters, all of which must be included in a mathematical model: (1) those parameters which are constant (e.g., based on the laws of nature); (2) those which are under consideration, often referred to as control variables, and values for which recommendations are to be made; and (3) "chance factors" which are not precisely known or controlled.[34] Random factors are either governed by the laws of probability or by enemy actions. In a more recent article Col. I. Pul'kin claimed that the required probability distributions were based on statistical data, but "in those instances where statistical data is lacking, it remains merely to postulate (predict) the values of random factors (parameters)."[35] While the purpose of operations research is to provide recommendations for parameters of the second group, consideration must be given to differing values for the other parameters so that the solution is optimal under any given condition.

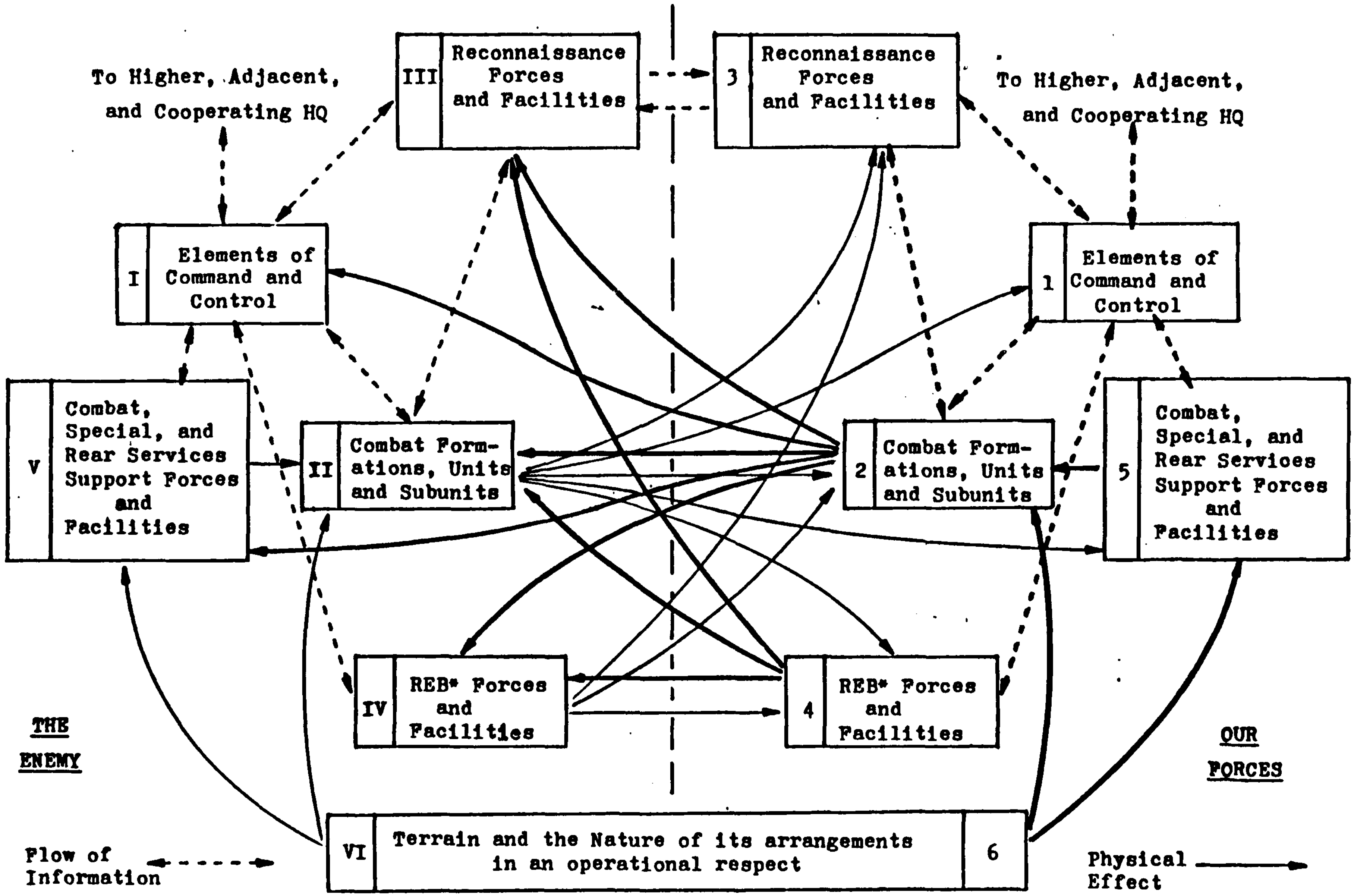
In addition, the appropriate "criterion of efficiency," which characterizes the degree to which the required mission is successfully accomplished, is defined at this point. The definition of the criterion of efficiency is easiest for weapons designed to destroy targets since it can be defined as the probability of destruction for point targets or the probability of damage for area targets. Weapons which provide different types of support for combat operations are more complicated and may require several criteria.[36] The output at this stage could be a block diagram such as that in Figure 2, which shows a possible model for combat operations of the ground forces.

In an article published in 1973, Col. F.K. Neupokoyev maintained that "combat capabilities" should not be confused with "combat effectiveness" when analyzing a military operation. Combat capabilities are determined by the characteristics of the weapons and military equiment and represent the "potential combat capabilities" of the forces.[37] The degree to which this potential is realized, or the combat effectiveness, depends on both the commander and his staff, as well as on enemy actions, which in general will not permit the realization of the full combat potential of friendly forces. The implication of this discussion is that combat effectiveness, as defined by Neupokoyev, is the apropriate criterion of efficiency in the modeling of combat operations.

As was noted, this first stage is where military science and mathematics are combined. This process requires a profound understanding of and experience in both fields. Consequently senior analysts such as Anureyev have called for the increased mathematical training of the Soviet officer corps through the introduction of a course in operations research at middle level and higher military academies.[38] Both Pevnitskiy and Moskvin in responding to this suggestion agreed with Anureyev, but noted that it would be equally important to increase the training of engineering and mathematical specialists in the area of operational art.[39,40]

In discussing this topic, Col. V. Ryabchuk contrasted Soviet and Western views on how operations research should be conducted and by

Figure 2. Example of structural diagram for subsequent development of algorithm
Source: Tatarchenko (1976; 10).
*REB - Radio-Electronic Warfare.



whom. He cited as an example of a typical Western view the opinion of Thomas L. Saaty who claimed that operations research should be performed by mathematical specialists within the constraints imposed by decision makers.[41] Lt. Col. Gusev, in reviewing the proceeding of a NATO conference on operations research, noted that the U.S. military tends to rely primarily on the use of civilian analysts rather than on training military officers in mathematical methods.[42] Ryabchuk strongly refuted the Western viewpoint since it violated the dialectical unity among goals, methods of achieving those goals, and utilizing the results of analyses. Gusev claimed that the West could never fully systematize operations research into a legitimate applied science. In particular, he strongly implied that in the West operations research could never be completely integrated into military science because the views of Western researchers were too fragmented.[43] These criticisms reflect the Soviet view that military science is itself a legitimate field of scientific research, analogous to the natural and social sciences. The Western practice of relying on civilian scientists who have little or no training in miitary art to solve military problems is seen by the Soviet military as absurd. To a Soviet military analyst it would be equally absurd for an economist to pose a problem in economics and then give it to a mathematician with little formal training in economics to solve.

In contrast, the Soviet practice is seen as clearly superior:

> In our case it will be the military specialist, who should possess appropriate experiences, possess a high degree of operational-tactical training, a certain minimum knowledge of mathematics and a clear idea of the content, capabilities, limitations, methods, techniques and forms of employment of operations research theory. Particularly important is good operational-tactical training, without which correct statement of research problems and successful

implementation of research results are impossible.[44]

The third stage, where the model is validated, involves the "most difficult methodological questions of modeling"[45] since full scale tests of the model's predictions are costly if not impossible. Validation is accomplishd by comparing the results of numerical calculations with expert estimates, the experiences of command-staff exercises and military games, and extrapolations of historical data. In addition pieces of the model which relate to the performance of military hardware can be verified by using the results of field tests.[46] Anureyev referred to this process as the "indirect practical verification" of the laws of armed combat, analogous to the practice of indirectly verifying the laws of natural science (e.g., the laws of astrophysics and cosmology where direct experimentation is impossible); he further maintained that this practice is employed in the military sciences "on an exceptionally large scale."[47]

A second feature of the model's validation is to insure its "workability," or that the required computation time is reasonable and that the results are sufficiently accurate. The level of mathematics should also be appropriate, especially for models intended for use by staff officers. The importance of this aspect of validation will be discussed in more detail below when actual applications are discussed.

The final stage in the development of a model is its introduction into use. While Soviet analysts appear to agree that mathematical methods do not provide firm solutions to military problems but provide only a more objective, scientific basis for making rational decisions, some disagreement appears to exist regarding the degree to which quatitative analysis can in principle solve military problems and replace the intuition and instinct of military commanders. Anureyev maintained that:

> Operations research must replace, and already is replacing, all subjective, irrational consideraions, such as intuition or instinct. ... However, operations research methods, as a rule, do not furnish the solution to the problem itself, but only give its quantitative numerical basis. One is not blindly guided by this basis, but takes into account a number of factors and concepts which cannot be expressed quantitatively (the moral-political factor, training, etc.), and only after this does he reach a final decision.[48]

Col. A. Tatarchenko expressed a similar view:

> However, it must be stressed that the results achieved by the methods of the theory of games (just as other purely qualitative measures) do not yet constitute the decision of the command element in the full sense of the word but they must be viewed as recommendations which can be adopted by him or rejected in the final evaluation of all qualitative and quantitative aspects of a developing situation in their totality.[49]

This viewpoint has been refuted by other authors who maintain that subjective considerations cannot be eliminated from military affairs, either in practice or in principle. For example, Bazanov and Malinovskiy in responding to the argument by Anureyev agreed that rough, qualitative evaluations should be replaced by more quantitative analyses, but intuition would continue to play a major role.[50] This criticism was reinforced by Moskvin who noted the limitations inherent in mathematical models: Not all factors can be included and some essential ones may be ignored. Consequently the final decision rests solely with the commanding officer.[51]

In an article published in PVO Herald, Lt. Col. Volkov noted that the capabilities of quantitative analysis will always be limited:

> It is only the commander who, by using all of the diverse information including that which does not lend

itself to machine processing, evaluates the combat situation, makes the decisions, assigns missions to his subordinates, and organizes the execution of his decision, the struggle of the personnel for victory. Victory cannot be calculated, it must be won. ... In other words, the advantages of a creative commander over an uncreative one will be even further strengthened and emphasized in ACS (automated control systems).[52]

Further evidence of the tension between quantitative analysis and commander intuition is offered by another article in PVO Herald where the authors cited the existence of confusion among the officer corps regarding the use of combat models. Some officers were reluctant to use mathematical models, believing that combat should not be modeled, but rather battle alternatives should be developed.[53]

An intermediate position was adopted by Tatarchenko in the definition of "modeling" in the Soviet Military Encyclopedia: models do not replace but supplement other methods, but they are becoming increasingly important because they reduce the degree of uncertainty and increase the scientific quality of decisions made. Consequently the main purpose of mathematical models is to obtain "an idea about to area of possible results and the distribution of the probability of their happening, about the tendencies of their changes and the relative cost of various versions of ideas, solutions and plans, or their individual elements."[54]

Ryabchuk noted that advances in operations research have demanded greater participation by man, even with the increased use of computers. He cited Charles Hitch that mathematics and computers are neither alternatives nor competitors with good intuition and judgement. Rather they are a means for preparing data for the decision maker, and simply provide another input. He also cited Edward S. Quade who maintained that judgement and intuition are often required for determining values for input parameters, even in engineering analyses. Ryabchuk then called for the increased role of the creative element in close unity with modern mathematical and logical methods.[55]

Mathematical models can also be used to help develop an officer's intuition during peacetime. Commanders who perform frequent calculations based on different conditions gain a familiarity with the possible solutions and outcomes so that in actual combat they need not perform all of the calculations; the results of previous calculations will be "remembered by officers like a multiplication table."[56] Given the revolution in military affairs and the "fact that military practice has not provided experience in the mass use of nuclear missile weapons and other modern means of combat,"[57] this application of operations research has become of increasing importance--although as of 1963 Anureyev maintained that it was underutilized.[58]

2.4 Classes of Mathematical Models

Different types of mathematical models can be distinguished by several factors, among which are the scale of application (tactical, operational, or strategic), the purpose (staff model, exploratory research, or troop/officer training), the means of construction (estimated, optimized, or "imitated"), and the mathematical devices employed (analytical, statistical, or combined).[59] The various types of models are summarized in Figure 3 on the next page. In models of higher level organizations, Tatarchenko reported that the following degree of detail, in terms of separate models for lower level units, is generally employed: for a front, models of divisions and "individual

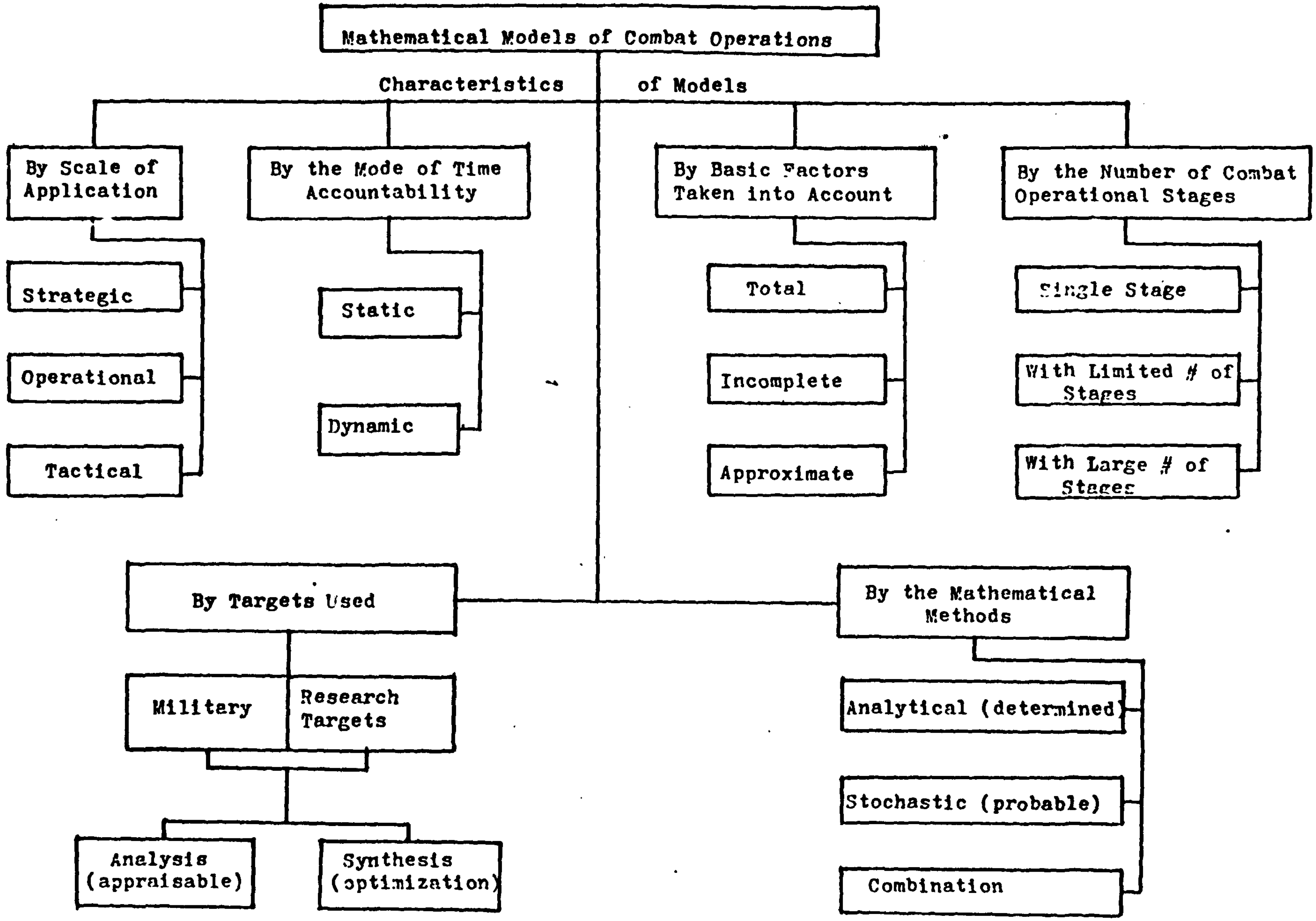


Figure 3. Types of Mathematical Models of Combat Operations
Source: Ionov and Beglaryan (1974; 10).

units" are included, whereas for an army, models of regiments and "individual subunits" are included.[60]

Anureyev distinguished between two basic methods of operations research: (1) statistical analysis which is based on data from past operations and is inductive in nature; and (2) mathematical prognostication which attempts to predict the outcome of future operations based on deductive analysis.[61] Statistical analyses were used in World War II, but are less relevant today given the radical changes in the nature of modern warfare. On the other hand, mathematical prognostication, which Anureyev referred to as the "richest contribution of the exact sciences,"[62] emerged only after the war--although some of his critics maintained that science has always been concerned with the problem of prediction.[63]

Statistical analyses were used to solve problems in air defense and anti-submarine operations during World War II. Its goal was to improve military operations based on statistical analysis of past operations which occurred under similar conditions. This method has several drawbacks: (1) a sufficient amount of data must be collected in order to reduce the level of uncertainty in the results; (2) the conditions of the various opeations must be sufficiently similar to justify the analysis (the lack of uniformity poses the greatest problem in statistical analysis, according to Anureyev); (3) much of the existing data is subjective in nature and collecting objective information is "still an unsolved problem."[64] In addition the collection of data for the analysis of modern warfare can be extremely expensive since a series of operations must be conducted, each purposely designed in a non-optimal fashion. Anureyev, nevertheless, maintained that statistical analysis is useful in the discovery, verification, and improvement of various kinds of recommendations in the course of military operations and in the quantitative analysis of past wars, war games, and command-staff exercises.

After the end of World War II operations research switched from the study of individual problems to the "complex evaluation of the effectiveness of weapon systems and even certain types of combat opeations."[65] More importantly, analysts were faced with a fundamentally new problem:

> In the post-war period operations research theory was confronted principally with problems of a new direction, linked with decision-making in the absence of any experience of similar operations, i.e., problems whose solution was possible only on the basis of scientific prognostication.[66]

Anureyev cited two examples where the methods of mathematical prognostication can be applied. The first example involved the allocation of nuclear weapons to targets whose characteristics were not known precisely; in addition, the available information and weapons may change during the course of the battle. Anureyev maintained that this problem may be solved at the level of a division without the use of mathematical methods, but at higher levels quantitative analysis was required in order to determine the optimal allocation with "strict regard and evaluation of every uncertainty."[67] The second example was the selection of modern weapon systems, where one cannot draw upon past experiences:

> Again prediction and prognosis are required. It is necessary to recreate a more probable picture of future military operations and make recommendations concerning a rational weapons system. This problem may best be solved by using the mathematical prognostication method.[68]

Anureyev further distinguished between two variants of mathematical prognostication: mathematical simulation and effectiveness evaluation. The difference between them is based on the extent to which the mathematical representation presents a complete picture of the operation under study. Simulation attempts to create a relatively complete picture with a certain degree of detail and is thus complex, requiring large computers. Effectiveness evaluation characterizes a military operation through the use of a "criterion of effectiveness," without establishing a complete representation. This second variant is simple and uses small computers, allowing solutions to be obtained quickly. Bazanov and Malinovskiy in their response to Anureyev's 1963 article maintained that this distinction was artificial because all analyses are based on calculating a measure of effectiveness.[69] Moskvin agreed with this criticism, adding that mathematical modeling "is not done as an end to itself."[70]

Mathematical simulation is further divided into two areas based on the mathematical methods employed. In analytical models, all quantitative relationships are represented by a series of analytical equations, e.g., Lanchester's equations. Pul'kin offered as an example of an analytical model the calculation of the effectiveness of a single strike on a point target. The probability of target hit with one round is given by[71]:

$$P = 1 - \exp(-(\rho R_p / B_o)^2), \qquad (1)$$

where R_p = the radius of the killing zone; B_o = "probably impact (burst) error from the mean point of impact (point of aim);" and ρ = a constant with the approximate value 0.477. For $B_o = R_p$ the hit probability is 0.2035, or approximately 20%. Pul'kin noted that: "With specific munitions power and type of delivery means, the effectiveness criterion can be increased by reducing ranges of projectiles."[72]

Random model simulations based on the Monte Carlo method, on the other hand, possess the distinctive feature of being experimental in nature, although they also have the drawback of requiring large, high speed computers. These models are used to reproduce tank battles and anti-aircraft engagements, to model target detection and destruction, and to analyze troops crossing rivers. Pul'kin noted that in practice the input data is limited to 10 to 20 or fewer data points, and that consequently estimates of only means and standard deviations are possible.[73] Anureyev claimed that "micro-models" have been used for small unit engagements and that "macro-models" "will be used to model regiments to divisions."[74] Monte Carlo simulations can be used in combination with analytical models:

> The random method, besides its independent worth, is also helpful in preparing analytical models and in checking the hypotheses brought forward in them.[75]

Anureyev recommended that mathematical simulations be used in large headquarters and in scientific institutions working on theoretical and practical problems in armament selection, tactics, and strategy.

In effectiveness evaluation a single or even several criteria of effectiveness may not accurately measure the success of a given military operation. Nevertheless the intent is "to correctly choose the most important criteria of effectiveness and to establish their connection with the parameters of the operation.[76] Although the method is only a rough approximation, it still can be useful:

> One may get only a tentative evaluation, but the approximateness and tentativeness of optimum solutions in this case is more than compensated for by

simplicity, and hence, the possibility of getting the necessary information quickly.[77]

Bazanov and Malinovskiy disagreed with this claim: these results are approximate and thus cannot be optimal. Furthermore, this view discourages the use of more precise methods only because they tend to be unwieldly.[78]

The following mathematical techniques have been applied to the quantitative analysis of combat operations according to Soviet analysts:

Probability Theory: Although probability theory is useful in analyzing military operations, Tarakanov cited difficulties in applying it in mathematical simulations; the calculations became greatly complicated.[79] Anureyev made a similar point, calling for simplifications in order to solve problems quickly through the use of tables, graphs, slide rules, and computer programs.[80]

Game Theory: Enemy actions can be modeled through the use of game theory, an application which will be discussed in more detail below.

Theory of Mass Servicing: Queuing theory is used extensively by Soviet analysts, particularly in the modeling of PVO forces--apparently both anti-aircraft and anti-missile defenses.[81] Col. A. Volkov maintained that queuing theory has been used with success in "evaluating the quality of available systems and modeling combat operations."[82]

Linear and Nonlinear Progamming: These tehcniques are used when the situation is known and does not change. Linear programming is "most commonly used to solve the problems of distributing targets and strike objects, as well as friendly forces and resources."[83] According to Anureyev, "many problems associated with the planning of the application of combat weapons (for example, if several weapons are allocated for one target) are reduced to problems on nonlinear programming."[84] While Anureyev's claim is technically correct, such problems can also be transformed into linear programming problems and solved more readily.

Dynamic Programming: When the situation is characterized by changing conditions and available data, then the techniques of dynamic programming are applied.[85]

Integral Programming: Since weapons can be allocated only in integral amounts, the techniques of integral programming are used to solve some target assignment problems: "For example, there are problems such as the target assignment of antiaircraft defense weapons, ground weapons of destruction, and others."[86] The "Hungarian method," a technique rarely used in the U.S. except to introduce the concept of linear programming, is used by the Soviets to solve the target assignment problem for air defense weapons.[87]

Stochastic Programming: In problems where "random factors play a substantial role," stochastic programming is employed. However it is useful primarily where expected values are meaningful (e.g., combat involving a large number of anti-aircraft missile complexes). In certain unspecified applications, it cannot be used because it produces "too many gross errors."[88]

Mathematical Modeling: The construction of overall models of combat operations in sufficient detail was not considered possible by Anureyev in 1966, due to inadequate mathematical techniques and limitations of available electronic computers. Consequently problems had to be broken down into components and solved separately.[89]

Network Planning: Volkov claimed that calculations using network planning revealed the possibility of shortening the time required for a number of control operations by 25-30%."[89] Network planning is also used in planning the repair and maintenance of complex equipment such as radars.

3.0 APPLICATIONS OF MATHEMATICAL MODELS

3.1 General Areas of Application

Mathematical models have been applied in the Soviet Union most successfully to troops armed with "mathematized" equipment, e.g., rocket, air defense, air, and naval forces.[91] The main areas of application, in order of increasing scale, are (1) evaluation of individual components of weapons systems and other equipment; (2) estimation of effectiveness of weapons systems as a whole; (3) calculation of the tactical capabilities of groups of forces; and (4) determination of optimal variants of combat operations.[92] The last two applications present the most difficulties. In estimating the effectiveness of a weapon system consideration must be given to interaction of man-equipment and men-weapon system, the effect of command, and unquantifiable factors such as troop morale and organization. The optimization of combat operations should also include enemy actions and account for outcomes which are influences primarily by maneuver and not fire-power.

Dmitriyev noted that one should consider not only the utility of mathematical modeling, but also the limitations in its application. In doing so, he differentiated among three different levels of the process of armed combat. The first or lowest level involves processes which are physical in appearance but military in character. Examples include the destructive means of nuclear, chemical, and conventional weapons and the transmission, reception, and processing of information. At this level abstractions are justifiable and "the effect is good."[93] The second level is specifically military in character and involves the actions of organized groups. This level includes processes such as the planning of military operations, troop control, and the maneuvering of forces and fire power on a battlefield. Many factors, both objective and subjective ones, play an important role at this level. "Yet even here known simplifications and abstractions of certain of the factors are permissable," and consequently this level is becoming more an object "for the application of mathematical methods which combine the concepts of operational research."[94] The third and highest level involves armed struggle in its totality and is too complex to be analyzed quantitatively. Mathematical models can have only an "auxiliary role." Dmitriyev concluded by writing that:

> The difficulties involved in a mathematical description of the processes of the armed struggle in all of its complexity suggests a stage in which simplified, partial, mathematical models of military actions can be built and investigated.[95]

As was mentioned above, mathematical models are used for several general purposes: research tool to advance military science, staff models for the analysis and planning of combat operations, the evaluation of troop training and command-staff exercises, and the evaluation of new weapons subject to cost constraints. In closing the 1963-1964 debate, Moskvin agreed with Venttsel' that Military Thought should publicize examples of the successful application of operations research.[96] However such examples did not appear until 1972. A note by the editors of Military Thought in August 1971 mentioned plans for the publication of some successful mathematical models:

> It is planned to publish a series of articles in this journal in the near future, containing a detailed discussion of contemporary analytical and statistical methods of military scientific research--Ed. Note.[97]

Examples of models used for these purposes are discussed below. Game theory is used for a variety of purposes, including weapon allocations (or as a staff model), evaluation of tactics (or for troop training and exploratory research), and weapon procurement. A second example involves the calculation of the combat readiness of military equipment, which can be used by staff officers to determine the ability of troops to accomplish their missions. The last two examples involve calculating the effectiveness of air defense forces. One model is based on the theory of mass servicing and is probably used by the staffs of large headquarters or by scientific institutes which analyze new air defense systems. The second air defense model employs considerable simpler mathematics and is thus intended for use by lower level staff officers.

3.2 Decisions Under Uncertainty

In an article published in Military Thought in 1973, Capt. 1st Rank Yu. Solnyshkov discussed the problem of evaluating possible decisions under conditions of uncertainty. He considered two forms of uncertainty: (1) uncertainty involving "random factors governed by specific, known laws," where uncertainty associated with missile accuracy is an example; and (2) uncertainty where it is "impossible to predict the probability of a given result" (or where the probability distributions are not known), with the behavior of the enemy being an example.[98]

In many cases the first type of uncertainty is handled by computing the expected value. However, there are important exceptions to this rule:

> In cases where actions are not repeated on a multiple basis and are aimed at performance of a vitally important mission, it is inadequate to utilize average results for a comparative evaluation of variants. It is necessary to guarantee mission execution. Then, for example, in calculating the requisite composition of weapons one specifies the probability of obtaining results not below a specific level and selects that variant of weapon composition (type and number) which

> will ensure fulfillment of this condition with minimum outlays. ... Determination of guaranteed mission execution is one of the methods of substantiating a decision with uncertainty of the first type.[99]

This philosophy indicates that Soviet military planners have adopted very conservative methods for accomodating the problems of uncertainty, particularly regarding missions of central importance.

The second type of uncertainty can be further divided into situations where uncertainty is not directly controlled by the enemy (e.g., the details of an attack scenario such as the weather and the terrain) and those where the uncertainty directly involves enemy actions. No general method exists for solving these problems since all solutions depend on the commanders relative aversion to risk. Solnyshkov presented several possible methods for solving problems where uncertainty is not due to enemy actions. The "maximin" criterion, which is based on *maximizing* the worst possible outcome, contains no element of risk and is a reflection of a cautious nature. The maximin rule assumes that if a decison maker must choose one of n alternatives D_j, the consequence of which depends on a random variable x, then x will take that value which results in the least advantageous outcome. The decision maker then picks the alternative D_j for which this least advantageous outcome is the greatest. This rule contains no element of risk because only the worst outcomes are used to determine which of the n alternatives is best; or one does not gamble on the possibility that anything but the worst could happen. While the maximin criterion may be too stringent a requirement for decision making in many tactical situations, it is applicable under certain conditions:

> In those cases, however, when actions are aimed at performing vitally important missions and it is

essential to ensure success under all possible conditions, the maximin criterion is the most acceptable.[100]

Other possible criteria described by Solnyshkov are minimum regret, maximax, Hurwitz (an intermediate criterion between the conservatism of maximin and optimism of maximax), and Laplace (which assumes all outcomes are equally likely). In general it is advisable to find a solution which is not heavily dependent on the methodology used and remains optimal regardless of changes in the situation.

In cases where uncertainty is based on the possible actions of the enemy, the problem can be analyzed using game theory. Tatarchenko maintained in an article in Military Thought that the mathematics of game theory possesses a correspondence to the principles of military science.[101] Both require one to assume that the enemy is intelligent and will not make mistakes. Surprise and the avoidance of routine are also found in both military strategy and game theory. "Mixed strategy" solutions provide an element of surprise by randomly alternating among two or more elementary strategies. The maximin criterion upon which solutions to zero sum games are based contained the virtue that it guarantees a minimum outcome regardless of enemy actions. "If the conflict situation is not thoroughly studied, in hoping without the necessary basis for the best of the theoretically possible results, one can be confronted in practice with the worst of them."[102]

Both Solnyshkov and Aureyev took a different position regarding the applicability of game theory, citing its extreme conservatism. Anureyev characterized this limitation by emphasizing the role of risk-taking and daring actions in combat operations:

In actual combat situations a reasonable risk is sometimes justified, decisiveness has great significance, and not all variants can be depicted as optimal for the sides.[103]

Solnyshkov discussed the limitations of game theory in more detail. In particular tactical applications where both sides possess roughly equal forces but where a decisive victory is required, the recommendations of game theory do not provide an adequate basis for making decisions; the expected outcome is a draw. In order to obtain a decisive victory, one must accept an element of risk:

> Under the condition of equality of force, an effort to attain decisive goals and the possibility of concealing actions from the enemy, one should not take this principle [maximin] as a guide. In these cases it is necessary to take a risk and pursue one of the action variants which provides maximum success. In studying the situation it is necessary to focus particular attention on action variants which appear improbable to the enemy.[104]

If the enemy is cautious, then it may be advantageous to deceive him into believing that one is employing the maximin solution when in fact one is not. The only situations under which the maximin criterion should be used are those where (1) all decisions are known in advance by the enemy so that deception is impossible; (2) it is essential to ensure victory (e.g., the use of "long range armament"); or (3) active, i.e., offensive, measures cannot be taken and the mission is to maintain the status quo.[105] If the Solnyshkov article (published in 1973) accurately reflects the attitudes of the Soviet military regarding decision-making under conditions of uncertainty, then the following implications can be drawn: (1) in performing calculations for strategic missions, such as the decision to declare war or the overall planning of military operations, the Soviet military uses extremely conservative guidelines to guarantee success; (2) in certain tactical engagements during the course of a war, Soviet commanders may be daring and take considerable risks to obtain decisive tactical victories.

Another area where the maximin criterion can be applied is the evaluation of arms development plans. Solnyshkov presented an example to illustrate this point: one is faced with the problem of deploying either ICBMs or SLBMs in order to upgrade the strategic arsenal. If only the "existing level of counterweapons by the opposing side" is considered, then the solution, as indicated in Table 4 below, is to purchase all ICBMs.[106] However, Solnyshkov maintained that this would not be adequate substantiation of the plans since it does not consider possible counteractions by the enemy. Instead the various combinations of resource allocations by "red" and the counteractions by "blue" should be evaluated based on the "average number of combat-effective missiles" available to "red." (It is interesting to note that here Solnyshkov used an average value, and not a conservative estimate as suggested above.) The maximin criterion is then used to determine the optimal mix of ICBMs and SLBMs: 40% of the available resources should be used on ICBMs and 60% on SLBMs guaranteeing at least 640 combat-effective missiles. The reason that this constitutes a reasonable solution is that decisions are known in advance by the enemy:

> Each side carefully observes the trends in development of manpower and military hardware of the other side and in conformity with this draws up and revises its own plans. Therefore the weapons system development plan should be elaborated taking into consideration the possible response reaction by the other side involving the development of counterattacking weapon systems, that is on the basis of the principle of maximin.[107]

Elsewhere Solnyshkov provided another condition when the maximin criterion should be employed in weapon development plans: "if the

Portion of resources allocated by Red for building ICBM	Portion of resources allocated by Blue for developing ICBM countermeasures 0	0.2	0.4	0.6	0.8	1.0
0	600*	620	660	690	720	760
0.2	620*	630	640	660	670	700
0.4	670	660	650	640*	640*	650
0.6	720	680	650	630	620	610*
0.8	740	700	680	640	620	590*
1.0	800+	740	700	680	620	580*

Table 4.
ICBM versus SLBM procurement problem
Source: Solnyshkov (1973; 56)

Notes:

* Minimum of each row, where the maximum of row minimums is 640 which corresponds to 40% of resources allocated to ICBMs

\+ Maximum -- if Blue deploys no ICBM countermeasures.

potential enemy possesses roughly equal or superior scientific-technical and economic potentialities."[108] In addition, non-zero sum game theory can be applied to the general problem of weapon procurement, with the possibility of cooperation being present.[109]

3.3 Combat Readiness of Weapon Systems

Capt. 2nd Rank V. Tsybul'ko described a procedure for the quantitative assessment of the combat readiness of weapon systems. The basis for determining the effectiveness of a weapon is its ability to perform its assigned mission, and understanding how combat readiness affects the ability is important. Tsybul'ko confined his discussion to weapons designed to "hit enemy installations" where the probability of target destruction is the criterion of effectiveness.[110] Only those factors which can be described quantitatively are included in the analysis: "the promptness of execution of combat missions," "the technical reliability of weapons and combat equipment," and "the faultlessness of crew actions."[111]

(1) Promptness: The factor of time is important in sucessfully accomplishing the objectives of a required mission; the execution time for a weapon system, T_{cm}, must be less than the critical time, t_{cr}. Many authors, such as Anureyev and Tatarchenko (an apparent reference to some analysis presented in Anureyev and Tatarchenko (1967)), treated the parameters as constants although both are in reality stochastic variables. If this method does not generate errors, it can be used; but under certain conditions, for example where the targets are missile submarines or mobile missile launchers, stochastic methods must be employed.[112]

Tsybul'ko discussed several methods for computing the probability, $P_n(t)$, that $T_{cm} < t_{cr}$. In general several tasks must be completed within the critical time, and the critical path method (CPM) system can be used to compute the distribution of job duration given lower, upper, and most probable estimates for the durations of each of the n stages. Alternatively statistical data can be collected for both the duration of each stage and the mission as a whole, and this data can be used directly. Calculations based on CPM possess "considerably more indefiniteness" than those based on statistical analyses and tend to be 15 to 25% less than the "true value" since the impact of noncritical paths is not considered.[113] Tsybul'ko claimed that complex calculations are required to compute $P_n(t)$ and that statistical estimates are thus preferred.

(2) Technical reliability: The criterion of technical reliability of an element of a weapon complex is given by: [114]

$$Q(t) = K_r P_{cm}(t), \qquad (2)$$

where K_r = the "coefficient of readiness" or the probability that the equipment is operational at any given time; $P_{cm}(t)$ = the probability of failure free operation during combat operations given that the equipment is reliable at the initiation of combat. This formulation implies an assumption of limited time to prepare for combat, or limited operational endurance, otherwise K_r would not be included since with adequate preparation essentially all equipment should be reliable at the beginning of the war. At the same time a prudent military planner would assume that readiness time is limited by enemy preparations to attack first.

This is a more stressful situation[115] that, nonetheless does not rule out a preemptive strategy.

Calculations of the technical reliability of equipment assume the statistical independence of the various subsystems, or that there are no common failure modes. The contribution of each element of a complex to the overall reliability can be one of three types: (1) failure of the element leads to a breakdown of the entire complex (e.g., a detection system or power source); (2) failure leads to the breakdown of a part of the complex (e.g. a set of interconnected missile launchers); and (3) failure affects only the ability to launch an individual missile.[116]

(3) Crew Operations: The use of complex equipment increases the importance of human operators. For example, human error accounted for[117]:

- 40% of the failures in missile tests,
- 63.6% of ship collisions, sinkings, and beachings,
- 70% of aircraft accidents (as reported by the U.S. Air Force).

No reference was given for the first two figures, or whether they reflected Soviet or Western experiences. In principle quantitative appraisal is possible for any activity, according to Tsybul'ko, but given the present state of knowledge only approximations are possible. Consequently, the criterion of reliability for operators during their watch can only be estimated using the following relationship[118]:

$$R(t) = K_r P_{we}(t), \qquad (3)$$

where K_r the coefficient of readiness, which is limited by a lapse of memory or distractions; and $P_{we}(t)$ = the probability of performing actions without undetected errors due to random loss of information or the intensity of incoming data. Tsybul'ko provided values for each parameter: K_r = 0.99 for receiving visual information and voice output; $P_{we}(t)$ = 0.97.[119] If there are k operators then the overall reliability R can be improved through redundancy, by, for example, having the performance of several operators checked by the commanding officer.

The combat readiness is characterized by the degree to which potential combat capabilities are achieved. The actual effectiveness is less than the potential due to delayed completion of tasks, weapon failure, and human error. The potential capability is given by:

$$W_m = 1 - (1 - W_1)^m \qquad (4)$$

where W_1 = the kill probability for a single missile and m = the number of missiles fired per target. The actual effectiveness, on the other hand, is given by[200]:

$$W_m(t) = P_n(t)\, Q_A R_A\, (1 - (1 - Q_B R_B W_1)^m) \qquad (5)$$

where Q_A and R_A are the reliabilities of equipment and men, respectively, for that portion of the complex needed for utilization of all m missiles; Q_B and R_B are the corresponding reliabilities for the launching of a single missile; $P_n(t)$ is the probability of prompt execution of all n stages of the combat mission. The effect of combat readiness is thus characterized by:

$$K_{cr} = W_m(t) / W_m, \quad (6)$$

where $0 \leq K_{cr} \leq 1$.

3.4 Mathematical Models of Air Defense

Anureyev in a 1967 article maintained that air defense is one area where mathematical models have been "successfully realized."[121] In 1973 two articles appeared in Military Thought, each of which presented a mathematical model for troop air defense. One article by Cols. P. Lozik and S. Petukhov was based on some relatively sophisticated mathematics and closely paralleled a textbook published later by Petukhov and A. Stepanov. The other article, which depended on considerably simpler mathematics, appeared to be designed for use by staff officers.

The Lozik and Petukhov article applied the theory of mass servicing to the problem of estimating the capabilities of air defense forces. The queuing system used in the analysis is characterized by M/M/n: Poisson arrival process, exponentially distributed service time, and n channels or individual servicing units. In general a queuing system is characterized by an input flow in which customers (e.g., aircraft) arrive at an average rate, λ_a. The customers are then serviced by one of n queuing channels (e.g., the aircraft are fired upon by one of n antiaircraft rocket complexes). Both the interarrival and servicing times are random variables characterized by probability distributions and average values. For the application to air defense, the problem is to compute the probability, P_{nf}, that all n channels are engaged while an aircraft is in range since then the aircraft penetrates the defense without being fired upon. This probability is used to compute the average target destruction probability[122]:

$$W = (1 - P_{nf})\, P_m K_{det}, \qquad (7)$$

where P_m = the probability the target is destroyed by m missiles and K_{det} = the detection probability. The detection probability, not discussed in any detail in this article, is considered to be an important parameter:

> Introduction of K_{det} is dictated by the necessity of taking into consideration the probability that a target will pass through the zone of effective fire due to the impossibility of firing on the given target and in view of insufficient time to take the requisite measures following detection, particularly under conditions of jamming or when the target is flying at a low or extremely low altitude.[123]

Given concerns of this nature, one might expect discussions of Western technologies used in electronic warfare to emphasize the resulting stresses on time constraints, and not, for example, on resource constraints (i.e., inadequate number of missiles). Calculation of W allows one to estimate the expected number of destroyed aircraft, which serves as a criterion of effectiveness:

$$M = N\,W, \qquad (8)$$

where N = the number of aircraft in the raid.

If, based on this calculation, higher echelon commanders decide that the air defense resources are inadequate, then they call up reinforcements or consider

> the taking of measures to diminish the enemy's capabilities (for example, by launching preemptive air and missile attacks against the offensive air weapon basing areas). In practice, however, one must deal with a deployed limit of air defense capability, and the task consists in utilizing the forces as efficiently as possible. We shall take this as a point of departure in our subsequent discussion, considering the most unfavorable conditions, where capabilities and number of weapons are limited, and we shall determine the optimal method of utilizing that which is available.[124]

Lozik and Petukhov thus adopt the philosophy proposed by Solnyshkov to handle the problem of uncertainty, namely the maximin criterion. An interesting point of comparison with U.S. models of air defense is that American analysts address primarily the problem of allocating air defense resources to defended targets, whereas Soviet analysts concentrate on modeling the dynamics of the engagement, taking the overall allocation as given. This difference could reflect the underlying views on military logistics: The Soviet "supply push" (where lower level commanders have no control over their supply rates) versus the American "demand pull" (where lower level commanders have considerable control over their supply rates).

The model is based on three input parameters:

(1) Air Attack Density, λ_a: The arrival process is assumed to be a Poisson flow where the probability that k aircraft arrive within a time τ is given by:

$$P_k(\tau) = \left(\frac{\lambda_a \tau}{K!}\right) \exp(-\lambda_a \tau), \qquad (9)$$

where λ_a is the average number of planes arriving per unit time (usually expressed in aircraft per minute). Lozik and Petukhov claimed that a Poisson flow is more difficult for air defense troops to handle than a regular flow; the effectiveness under more favorable conditions would thus be better.[125] The use of a Poisson flow is an example of the application of the maximin criterion. The value for λ_a is calculated as follows:

$$\lambda_a = N \ V_t \ / \ D_a, \qquad (10)$$
$$= N / T_a,$$

where N = the number of attacking aircraft; V_t = the target speed; D_a = the depth of the aircraft formation; and T_a = the duration of the attack which is "usually specified."[126]

(2) Service Time, t_f: A fairly complex mathematical calculation is required to estimate t_f, which involves target detection time, time to fire each missile, missile flight time, time to shift fire, and time between firings. Alternatively one can estimate the number of missiles that could be fired during the attack, q, and calculate t_f as follows:

$$t_f = T_a / q. \qquad (11)$$

(3) Waiting Time, t_w: For those missile complexes that have a large effective fire zone arriving aircraft need not be "serviced" immediately (i.e., if the complex is "busy" when it arrives). The plane will remain within range for a time t_w:

$$t_w = (D_r - d_r) / V_t, \qquad (12)$$

where D_r and d_r are the distances to the outer and inner boundaries of the effective fire zone at the plane's altitude, respectively.

Lozik and Petukhov distinguish between two types of antiaircraft rocket complexes: (1) those with a small effective kill zone where arriving aircraft pass through unattacked if all air defense channels are "busy" when they arrive ($t_w = 0$); and (2) those with a large effective kill zone where arriving aircraft "wait" for a limited amount of time ($t_w > 0$) before leaving the kill zone unattacked.

For n single channel complexes with small kill zones, the probability of delivering fire on a target is given by[127]:

$$P_f = 1 - P_{nf},$$

$$= 1 - \frac{\frac{\alpha^n}{n!}}{\sum_{k=0}^{n} \frac{\alpha^k}{k!}}, \qquad (13)$$

where $\alpha = \lambda_a t_f$. A derivation of this equation is provided in an appendix below along with some sample calculations. Lozik and Petukhov provided an example of the use of this equation, but claimed that "all figures characterizing quantitative and qualitative composition of air defense forces and enemy attack forces are arbitrary."[128] In the example, $N = 24$, $n = 3$, $P_m = 0.5$, $K_{det} = 0.7$, $t_f = 0.5$ min, $\lambda_a = 4$ aircraft/min.[129] Thus $\alpha = 2$ and

$$P_f = 1 - \frac{2^3/3}{1 + 2 + 2^2/2 + 2^3/3} = 0.79;$$

$$W = (0.79)\ (0.5)\ (0.7) = 0.276;$$

$$M = (24)\ (0.276) = 6.6$$

In this example only 6.6 out of the total of 24 attacking aircraft are destroyed.

For complexes with a large kill zone, the Venttsel' formula is used[130]:

$$P_f = 1 - \frac{\beta}{\alpha} \; \frac{\dfrac{\alpha^n}{n!} \displaystyle\sum_{s=1}^{\infty} \frac{s\alpha^s}{\prod_{m=1}^{s} (n + m\beta)}}{\displaystyle\sum_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^n}{n!} \sum_{s=1}^{\infty} \frac{\alpha^s}{\prod_{m=1}^{s} (n + m\beta)}} \qquad (14)$$

where $\beta = t_f/t_w$. A derivation of this formula is given in the appendix along with a discussion of the assumptions behind it and some sample calculations. Given the obvious complexity of this equation, the authors provided a short table of values for P_f. A second example using the Venttsel' formula was given, where N = 24, n = 3, $P_m = 0.9$, $K_{det} = 0.9$, $t_w = 1$ min., $t_f = 0.5$ min., and $\lambda_a = 4$ aircraft/min.[131] Note $\alpha = 2$ (as above) and $\beta = 0.5$, so that

$$P_f = 0.82;$$

$$W = (0.82)\ (0.9)\ (0.9) = 0.66;$$

$$M = (0.66)\ (24) = 15.8.$$

(In the article the value for M was given incorrectly as 13.9, although this could be a translation error.)

The authors also presented an example where the optimal echelonment in depth of two air defense lines consisting of two different types of missile complexes was calculated. The problem was to determine which complex to put first[132]:

> Variant No. 1: First echelon consisted of high altitude, rapid fire missile complexes with small kill zones and $P_m = 0.9$, $t_f = 0.5$ min.; second echelon also consisted of missile complexes with small kill zones but with $P_m = 0.8$ and $t_f = 1.0$ min.
>
> Variant No. 2: The same two types of missile complexes but in the reverse order.

For both air defense systems $K_{det} = 0.9$; $\lambda_a = 10$ aircraft/min. and $N = 50$. Table 5 below summarizes the results obtained by Lozik and Petukhov, along with some corrections. Since the table was included in the original Russian these mistakes cannot be attributed to translation errors. The corrections do not, however, substantially alter the conclusion: the first variant is still optimal.

The second model, based on an article by Gen. Maj. M. Botin and Lt. Col. P. Ivankov, is intended to help establish the air defenses needed to achieve a given level of effectiveness. A possible criterion of effectiveness is "averted losses," defined as "the losses hostile aircraft could have inflicted on defended troops or other targets"

Table 5. Sample Air Defense Calculations
Source: Lozik and Petukhov (1973; 85)

Characteristics	Variant No. 1	Variant No. 2
(Value for α_I)	(5.0)	(10.0)
Effectiveness of anti-aircraft weapons on first line, determined with formula	(0.93)(0.9)(0.9)	(0.66)(0.8)(0.9)
	= 0.75	= 0.475
$W_I = (1-P_{nI}) P_{mI} K_{det}$		
Mathematical expectation of number of aircraft penetrating first air defense line	12.5 (2.5*)	5.2 (5.3*)
(Value for α_{II})	(2.5)	(2.6)
Effectiveness of second line:	(0.1)(0.8)(0.9)	(0.92)(0.9)(0.9)
	= 0.72	= 0.74
$W_{II}=(1-P_{nII})P_{mII}K_{det}$	((0.997)(0.8)(0.9)	((0.996)(0.9)(0.9)
	= 0.72**)	= 0.81**)
Number of aircraft penetrating second line	3.5	8.9 (5.1)
Number of aircraft destroyed by entire air defense	46.5	41.1 (44.9)

Notes:

(. . .) = corrections or additional information.

* $\lambda_a = N_I / T_a$, where $T_a = N \ /\lambda_a$ = 5 min and N_I= the number of aircraft penetrating first line.

**The corrected values for P_{nII} were computed using α_{II} and n (the number of channels) = 8; note that the values for P_{nII} are consistent with n = 8.
The "0.1" is probably a typographical error, but the "0.92" used in Variant No. 2 corresponds to α_{II} = 5.2 (or the value for λ_{aII})

(without the presence of the air defenses).[133] Calculating this number under battlefield conditions is too difficult for the procedure to be practical. Thus the authors presented an alternative method based on the air defense reliability factor[134]:

$$K_{PVO} = M(c) / N_{ts}, \qquad (15)$$

where M(c) = the expected number of destroyed enemy aircraft and N_{ts} = the number of air targets in the attack or during a period of operations. The authors claimed that this formulation has been validated using statistical data from World War II, the Great Patriotic War, and local wars. This analysis has shown that air attacks were either broken off or had their effectiveness greatly reduced when losses exceeded 15 to 35%, or as high as 40%.[135]

> Hence we conclude that air defense of troops and other targets is sufficiently reliable if the reliability factor of the air defense forces (K_{PVO}) is equal to or greater than the air defense reliability factor at which enemy aircraft refrain from completing their combat mission (K_{ot}).[136]

The required air defense reliability factor is given by:

$$K_{PVO\ TR} \geq K_{OT} / (1 - K_{PT}), \qquad (16)$$

where K_{PT} is the loss factor based on the expected losses of air defense equipment under specified conditions.

In order to calculate K_{PVO} and compare it to the required value, $K_{PVO\ TR}$, the value of M(c) must be found[137]:

$$M(c) = N_o \, m_{STR} \, P_P \, K_U \, K_{VCh}, \qquad (17)$$

where: N_o = the equivalent number of antiaircraft weapons of a given type; m_{STR} = the number of rounds fired; P_p = the target hit probability; K_U = the fire control system reliability; K_{VCh} = the coefficient for the participation of antiaircraft weapons in repulsing the attack. If several types of antiaircraft weapons are used, then the total number of destroyed aircraft is:

$$M(c) = \sum_{i=1}^{n} M(c)_i, \qquad (18)$$

where $M(c)_i$ is the number destroyed by the ith type of antiaircraft weapon.

The five parameters needed to calculate M(c) are described below:

(1) N_o is defined using the concept of "conventional target channels" (CTC), an approximate analogue to the U.S. Army's "fire power scores" (FPS). A CTC is defined as "the aggregate of antiaircraft weapons capable of independently detecting and firing at an air target, with a specified hit probability."[138] Several antiaircraft complexes might be included in one CTC unit in order to achieve the given hit probability. Note that a low CTC score is "better" than a large score since fewer weapons are required. Botin and Ivankov claimed that the use of CTC scores enabled one "to make different types of antiaircraft complexes commensurable in regard to the most important characteristic -- probability of target hit,"[139] and thus greatly simplified calculations. N_o is the number of CTC units of a given type of air defense weapons,* and all other parameters are expressed per CTC unit.

*Note that if it takes 5 units of a given type to comprise one CTC unit with P_p = 0.5 and 10 units are available, then N_o = 2.

(2) The total number of rounds of ammunition fired is given by[140]:

$$m_{STR} = S_n / S_{ts}, \qquad (19)$$
$$= T_n / T_{ts} + 1,$$

where: S_n = the average number of missiles or amount of ammunition fired during the duration of the attack; S_{ts} = the average number of missiles or amount of ammunition fired at a single target; T_n = the duration of the attack; T_{ts} = the length of a firing cycle..

(3) The hit probability per CTC is defined as:

$$P_p = 1 - (1 - P_1 K_{pr})^n, \qquad (20)$$

where P_1 = the kill probability for one missile or antiaircraft weapon; n = the number of missiles or weapons fired in one CTC; and K_{pr} = the "coefficient taking into account all types of hostile air activity against air defense weapons."[141] K_{pr} could be an attempt to account for the effects of enemy electronic countermeasures (ECM) since combat losses are presumably included in K_{pT} in equation (16) above.

(4) The probability of successful fire control, K_U, achieves its highest value with automatic means of control and its lowest value with the "plotting-board control arrangement."[142] Detection is an important factor in fire control: if the distance at which targets are detected is less than that required for successful control the decentralized fire control should be instituted.

(5) The participation coefficient is computed as follows[143]:

$$K_{VCh} = K_{bg} (1 - K_m) K_{BCh}, \qquad (21)$$

where K_{bg} = the combat readiness factor, or the fraction of the total CTC score ready to engage the enemy; K_m = the maneuver factor, or the fraction unable to participate in the attack due to maneuver (presumably to increase survivability); K_{BCh} = the probability that the target enters the fire zone of the antiaircraft complex. If there are a sufficiently large number of PVO weapons distributed uniformly, then:

$$K_{BCh} = (N_n/N_{ts})\ (2\ M\ P_{pr} + \sum_{j=1}^{M} L_j)\ /\ L_F, \qquad (22)$$

where M = the number of directions of attack; P_{pr} = the "maximum CTC course parameter;" L_j = the width of the attack front in the jth direction; L_F = the width of the total attack front; N_n = the number of air targets within the effective altitude of the antiaircraft weapons. For defense of troops or other targets:

$$K_{BCh} = (N_n/N_{ts})\ (2\ P_{pr} + I)\ /\ L_F, \qquad (23)$$

where I = the average distance between equivalent antiaircraft complexes with unit CTC scores.

This method can be used to calculate the effectiveness of established air defense forces, determine the required number of weapons to achieve a given $K_{PVO\ TR}$, and establish the acceptable level of air defense losses while still maintaining a specified reliability factor. For example the required number of antiaircraft complexes, expressed in CTC units, for a specified reliability factor is[144]:

$$N_{ZK} = \frac{K_{PVO\ TR}\ N_{ts} - M(c)}{m_{STR}\ P_p\ K_U\ K_{VCh}} \quad . \tag{24}$$

For this method to be practical, it would appear that manuals listing many of the above parameters would be required.

4.0 EVIDENCE OF THE USE OF QUANTITATIVE ANALYSIS

In 1973, Tarkanov claimed that:

> Until just very recently the very possibility of simulating troop operations as an integral phenomenon was doubted by many researchers. Moreover, many of the attempts to simulate operations at the start were essentially unsuccessful.[145]

The advancements in military science, the increased experience in simulating combat operations, and the advances in mathematics and computers have enabled Soviet analysts to come close to solving this problem. Prior to 1972, all of the articles on mathematics and operations research appearing in Military Thought discussed only the general applicability of quantitative methods to military affairs. However in 1972 and 1973 (the last two years for which Military Thought is available publicly) a change appears to have occurred: five articles each presenting specific mathematical models with sample calculations appeared -- perhaps as evidence to Tarakanov's claim.

In the 1963-64 debate two articles called for the publication of an "overt journal for operations research" which would also include translations of articles from the U.S. journal Operations Research.[146, 147] One article suggested that during the initial period before such a journal was established a supplement to Military Thought could be published. Although no actions appear to have been taken on these suggestions, their appearance in Military Thought at least indicates interest in disseminating articles on operations research more widely.

Only indirect evidence that Soviet military analysts use mathematical models can be found. Nevertheless, such methods are apparently used in the evaluation of troop exercises, in weapon procurement plans, in evaluating the effectiveness of new and existing weapon systems, and by staff offficers in planning combat operations. Special attention is given below to evidence that Lanchester's equations are employed in the modeling of ground forces.

4.1 Evaluation of Troop Exercises

Lozik and Petukhov maintained that one application of their air defense model is the evaluation of PVO exercises:

> For example, determining how and when the decision was made to destroy the means of air attack in conformity with the "air attack" plan, and then synthesizing these data, the umpires can compare them with the results obtained from the corresponding formulas and can draw better substantiated conclusions on the level of preparation of those units taking part in the exercise.[148]

Further evidence of this application is given by Anureyev:

> In the national air defense troops it is possible to create a situation sufficiently approximating an actual hostile air attack, and one can obtain a number of important theoretical generalizations on the effectiveness of air defense forces on the basis of a few particular results, employing mathematical methods. The same applies to other branches of the armed forces, for which indirect practical results serve as an important criterion for determining the correctness of theoretical propositions.[149]

Mathematical models are cited as being useful in evaluating training exercises:

> Further, the widest possible utilization of mathematical models is necessary during the conducting of games and training exercises both with the application of electronic computers as well as without them.[150]

4.2 Weapon Procurement

In reviewing a book by Yu. V. Katasanov on the American use of PPB

(Planning, Programming, and Budgeting) in the Pentagon, Solnyshkov revealed Soviet interest in using quantitative methods. "He [Katasonov] particularly stresses that at present it is impossible to insure the effective use of resources allocated for military needs without the use of mathematical economics methods and above all the methods of systems analysis."[151] Although Solnyshkov thought that this book could be useful in working out practical problems, he criticized its emphasis on the organizational aspects of PPB which were intended to reduce the effects of interservice rivalries.

> [PPB] is nothing fundamentally new for specialists familiar with mathematical economics methods. In our country special programs are worked out regularly. For example, the agricultural development program, etc. . . . In the process of working out such a program extensive use is made of modern mathematical economics methods, including the methods of the theory of operations research and systems analysis. In this regard we feel that the value of the work would have been greatly increased if the author had been able to bring out the basic concepts in the methodology of systems analysis used as the quantitative basis for decisions in the PPB system.[152]

In other words, Soviet analysts already understand how to solve organizational problems. The Katasonov book would have been more useful to them if it concentrated on the integration of quantitative methods into the planning process -- a problem, which this criticism implies, is of interest to the Soviet military.

Ryabchuk discussed the use of systems analysis by American scientists in evaluating the cost effectiveness of modern weapons systems by noting that, in contrast to the period immediately following World War II when the U.S. military used "formal optimization" or "nothing but the best" as a requirement, more recently less stringent requirements have been applied:

> With the coming of the "atomic age" the cost of weapons development increased astronomically, and this approach became unacceptable. It was gradually replaced by another: "Only that which is essential, and at minimum cost."[155]

New techniques were required in order to analyze complex systems as a whole; these techniques comprise the field of systems analysis which has been applied primarily to long range military planning under conditions of uncertainty. The use of quantitative methods by the U.S. military was also mentioned in some comments at the end of Gusev's review of a NATO conference (it is not clear who wrote these comments, Gusev or the editors of Military Thought, since they are clearly separated from the article itself). The U.S. military made use of operations research in its decision to deploy a submarine missile fleet and not strategic aviation, ICBMs, or carrier-based aircraft. Given the role of major corporations involved in weapons production, it was not easy for a capitalist country to change procurement plans.[154] Although both discussions were based on American use of mathematical analyses, it is possible that they also reflected Soviet experiences and interests. For example, the second example could have been intended to demonstrate the utility of operations research in overcoming bureaucratic pressures to deploy weapons of limited military value.

4.3 Weapon Effectiveness Evaluation

Anureyev maintained in a Military Thought article published in 1963 that game theory has found "wide application in the analysis of the efficiency of weapon systems."[155] Perhaps one such application was the calculation of the correlation of nuclear forces as proposed by Anureyev in 1967, where game theory could be used to determine the optimal allocation of nuclear weapons.[156] Anureyev was, however, criticized for the complexity of his proposed formula for the correlation of nuclear forces, a difficulty which made his methods impractical.

> . . . it is practically impossible to collect in the time required that great quantity of initial data on friendly troops and especially about the enemy which is required to make the calculations proposed by the author for calculating the correlation of forces of belligerents at each moment.
>
> ***
>
> At first glance it [the Anureyev equation] creates a favorable impression by its simplicity, but after a closer look it is evident that the simplicity is illusory.[157]

This critic, Col. L. Semeyko, claimed that the Anureyev equation could find application for strategic scale calculations before a war starts since the time factor is less important, enabling one to make the detailed calculations required, and the initial data changes slowly. It would not be expedient to extend this formulation to the operational-tactical scale where the requirements are maximum speed and simplicity. Given that Semeyko called for the development of a "more limited interpretation" of the correlation of forces[158] -- one perhaps more applicable to the operational-tactical scale -- a reasonable conclusion is that this criticism did not question the principle of using numerical assessments of the correlation of forces. The difficulty may have been that Soviet military officers wanted to employ such methods, but this particular one proved to be unworkable, which is a key ingredient in the validity of a mathematical model.

Quantitative analysis is "most fully applied in missilery"[159]:

> The combat capabilities of nuclear-missile weapons, computed theoretically on the basis of range and proving ground tests, are receiving further

> conformation in tactical combat training on the basis of particular, indirect results (combat readiness, accuracy, reliability, performance characteristics).
>
> On the basis of the laws of similarity, which was experimentally established for conventional munitions, one can predict the combat capabilities of nuclear weapons of a megatonnage which has not been actually tested.[160]

The "laws of similarity" are clearly the scaling laws where nuclear weapon effects scale with the cube root of the yield. Quantitative analysis is also used to compute the expected damage to targets and optimal missile strikes.[161] In addition, increasing the effectiveness of nuclear missiles received attention:

> The problem of increasing missile accuracy requires utilization of the results of many of the natural sciences: ballistics, theory of probability, geodesy, celestial mechanics, theory of potential, and others. Of importance for further increasing the effectiveness of ballistic missiles are studies pertaining to warhead design and self-contained control systems.[162]

The reference to "self-contained control systems" indicates Soviet difficulties with the guidance systems of their ICBMs, and particularly with inertial guidance systems capable of correcting unexpected errors in flight.[163] The contribution of material sciences to improvements in heat shield materials, another important factor in improving accuracy, received no mention in the above citation.

4.4 Staff Models of Combat Operations

In reviewing the book Application of Mathematical Methods in

Military Science by Anureyev and Tartarchenko, Gen. Maj. N. Smirnov and Col. Bazanov praised it by writing:

> The value of the material in the book under review is characterized first of all by the fact that the recommendations given by the authors can be used in the work of staffs at the present time without any particular difficulty.[164]

Most of the criticism given in the review centered on the need for more illustrative examples and more detailed discussion of mathematical methods.

> Undoubtedly the value of the second chapter and the interest in it would be greater if there were more examples of application of the theory of probability in evaluating the effectiveness of calculations associated with the combat use of combined arms soyedineniye. This is all the more so since despite many difficulties certain definite results have already been achieved in this area.[168]

The limited size of the book restricted its ability to "describe models which are already found in application in military affairs."[166] The reviewers thought that more attention should have been given to the theory of mass servicing as well as other fields of mathematics.

> Unfortunately, many mathematical methods which have found effective application in military affairs during the past few years such as methods of network planning and control, the theory of search, the theory of seeking solutions, and others are not treated in the book.[167]

Consequently then Smirnov and Bazanov called for a new edition to be written and made available in larger numbers.

Ryabchuk, writing in 1971, noted that much needed to be done in the area of operational-tactical problems, citing as examples of useful works the Anureyev-Tatarchenko book and one by Yu. Chuyev (Operations Research in Military Affairs, Voyenizdat, 1971). The main trend should be the study and adoption of operations research to the work of commanders and staffs of all echelons.[168]

4.5 Applications of Lanchester's Equations

One important class of mathematical models which have received special attention by Soviet analysts are those based Lanchester's equations. Evidence that these equations are used by the Soviet military is provided by Anureyev who wrote in 1963 that:

> Even in simplest form Lanchester's equations prove to be very beneficial in obtaining recommendations for the solution of a number of the most important operational and tactical matters ... [169]

Writing in 1972, Anureyev claimed that Lanchester's equations were used in scientific studies of the art of warfare: "For example, Lanchester's equation is frequently employed in analysis of combat by motorized rifle and tank units."[170] In his correlation of forces article, Anureyev used the results of Lanchester's equations to derive the correlation of tank forces.[171]

Soviet analysts do, however, note that these models possess certain limitations. In the Soviet Military Encyclopedia, Tatarchenko wrote:

> Such models were suitable only for the most general, abstr .t conclusions and recommendations, however, the basic ideas incorporated in them (taking into account the dynamics of combat actions, their two-way nature, the interrelationship between the quantity and quality of combat units, taking into account the entrance of reserves, etc.) have been used many times in the development of contemporary models.[172]

One important limitation, cited by Anureyev, is in combat where the outcome is determined more heavily by maneuver than fire power, e.g., in tank operations. In such cases, Anureyev called for:

> special attention to the development of such mathematical methods which would permit the analysis of combat operations of troops, and above all of the

ground forces, where the problems are solved by maneuver, movement, and less by fire-power.[173]

Anureyev alluded to the possible application of non-zero sum game theory to tactical scale models of ground forces, although not necessarily to address the effect of maneuver.[174]

5.0 CONCLUDING OBSERVATIONS

Mathematical modeling for military planning attracted the interest of military specialists in both the U.S. and the U.S.S.R. in the early 1960s. Many of the military applications of mathematical modeling in both countries, then and now, are quite similar. Yet, it would be a mistake to assume that mathematical analysis has affected, and has been affected by, military planning in the two states in similar ways. Our effort to better understand Soviet military planning through the window of their mathematical modeling work, leads us to conclude that the Soviet experience has been quite different from that of the United States. There are three groups of factors that seem particularly relevant: structural considerations, substantive emphases, and functional applictions.

5.1 Structural Considerations

The first consideration has to do with the nature of the Soviet military establishment itself. The policy analysis and policy making organs of the Soviet Ministry of Defense are populated entirely by professional military officers--in contrast to the highly civilianized U.S. Department of Defense. All deputy ministers of defense, all their assistants, all program managers, etc. are active duty military officers. There is only one ranking civilian who is occasionally placed within the Ministry--the Minister of Defense who is selected by the Politburo--and even he is given the military rank of Marshal of the Soviet Union upon assumption of office. There are no Soviet defense "think tanks" teaming with civilians eager to provide analytic support. The advisory role of civilian scientists and engineers who work in the military sector, while crucial, is highly circumscribed and limited to military technical analysis of weaponry within their given areas of exertise. Force posture planning and military analysis, are conducted and implemented almost entirely by the professional military.

It is within this setting that mathematical modeling enters Soviet military science. The tools of operations research, systems analysis, computer simulation and modeling were introduced by the Soviet professional miltary for the professional military. The locus of this mathematical modeling work within the Soviet military establishment is rooted in the General Staff, the various service staffs, and the myriad military academies. Research and publication on the applications of mathematics to military affairs are authored by officers with ranks of colonel to general-colonel. Most, if not all, also hold equivalents to Ph.D.s in some branch of science or engineering. These officers develop the analytic techniques, devise computer codes, collect and organize the data, and carry out the analyses that support Soviet military decisionmaking. Here we are talking abut large combat evaluation models, nuclear exchange models, mission-cost effectiveness models, and military economic evaluation models; not simple engineering tactical-technical models (e.g., the aerodynamics of cruise missiles).

Soviet specialists argue that mathematical modeling is an important tool for producing "objective" military analysis. In fact it does serve to insulate Soviet military planning from the type of capricious civilian interference that occured during the Khrushchev era. In this respect their embracing of mathematical analysis was directly opposite the U.S.

experience in which civilians brought systems analysis to the Pentagon in the hope of "reigning in" the armed services.

5.2 Substantive Emphases

Since Soviet specialists in the military applications of mathematical modeling are all fairly high ranking military officers they acquire expertise in military art (strategy, operational art, and tactics) and military science before developing their skills in mathematical analysis. This is the reverse of the U.S. experience, where civilian specialists in mathematics, computing, and operations research arrive at the Pentagon, the CBO, or the GAO with little or no prior military knowledge, and certainly without high level professional military experience. Indeed, as was described earlier this difference has been noted by Soviet observers. They strongly argue that their approach is superior since Soviet analyses are structured by military art and military science, and then mathematical tools are applied. In the U.S. approach the mathematical tools structure the analysis, and then military problems are studied. Sidestepping the issue of which approach is preferable, it remains that the difference could well have a significant impact on the conceptualization and formulation of military analytic problems and mathematical models.

There is also a difference in the mathematics base from which the two countries operate. The Soviets work with many statistical, optimization, and probability models that are peculiar to their research in mathematics. A substantial amount of this work--civilian as well as military--is based on statistical theory that has not circulated widely in the West.

The differences in the mathematics bases of the two countries is further accentuated by the relative lack of computers in the Soviet Union. There is considerably less "number crunching" in Soviet modeling work and much greater recourse to theoretical development and analytic approximation. Thus, Soviet mathematical modeling studies can--and do--reach conclusions that differ from those of American studies, even though the databases may be similar.

Soviet outcome indices (i.e., those variables that they are interested in optimizing or measuring) can differ markedly from those common to U.S. studies. In particular there is a startling Soviet obsession with "time" as a critical outcome index in mathematical analyses. That is to say, mission accomplishment (or a certain level of mission effectiveness) is assumed and the analytic issue is: How long does it take to successfully reach that level of mission accomplishment? This interest in time is amply reflected in air defense analyses (and by extension ballistic missile defense analyses) which are structured as mass servicing problems. All improvements in air defense capabilities, whether technical, operational, or "human," are ultimately reduced to time indices. For within the mass servicing framework reductions in service times or increases in the length of time a target can be kept in the service queue automatically produce increase in mission effectiveness.

Indeed, the Soviets appear to use time as a common metric in comparisons of dissimilar methods of accomplishing the same mission.

5.3 Functional Applications

Beyond simple tactical-technical analyses Soviet work in mathematical modeling seems to be concentrated in four areas of study:

cost-effectiveness, combat analysis, combat readiness, and automated troop control and weapons system control.

As one might expect, some Soviet functional applications of mathematical models are quite similar to those found in the U.S. Defense Department. Soviet cost-effectiveness studies, among all Soviet modeling efforts, are perhaps the most similar to American modeling work. The problem is the same on both sides of the globe: How does one choose among competing weapons systems within a given mission area? Yet, even while the nature of the problem is the same, as is the objective of the analysis, the indices of effectiveness often differ. (See our first report for a detailed discussion of Soviet cost-effectiveness studies.)

Soviet modeling of ground force combat operations tend to follow lines parallel to U.S. Army efforts. They employ Lanchester-type models, stressing asymmetries in quantitative capabilities over asymmetries in qualitative capabilities. In studying large unit engagements efforts have been directed towards devising systems of effective firepower equivalents to cope with the problem of heterogenous forces. Thus, only the most macro performance capabilities of weapons systems are incorporated in these studies. These include: lethality, delivery rate, accuracy, reliability, survivability, command and control responsiveness, troop control, and quantity. Air defense models follow a very different mathematical path. They are built on queuing theory (mass servicing). While an analogue to firepower equivalents is sometimes employed here to ease computation, these models do allow for greater separation of technologies. This enables Soviet specialists to examine the impact of alternative deployment schemes, varieties of tactics, and new weaponry. As far as we can tell, this analytic flexibility has not been achieved in the ground force area.

Much more unique to the Soviets is their modeling work on combat readiness and automation of troop control. In both cases, their intent is to improve mission effectiveness through the reduction of "control times." Modeling studies of combat readiness are often contained in weapon effectiveness analyses, thereby integrating the human and technological aspects of modern weapons systems. Time optimization is another objective of combat readiness modeling efforts. How and where can the components of control time be reduced so as to increase the apparent effectiveness of a given weapons system? For example, changes in the staff procedures at radar command posts might cut 15% off of time required to handoff information to firing batteries. This, in effect, would increase the effective firepower and lethality of the battery.

Modeling work related to the automation of troop control is a more direct attempt to cut control times. Soviet discussions consider several levels or stages of automation. The simplest application involves the use of computers to help to organize and manage command staff work. Mathematical models aid the automation system by carrying out necessary calculations; controlling necessary databases; controlling the flows of information among observation, command, and operations units; and improving the efficiency of planning efforts. At the other extreme, there is an ultimate hope the combination of large accessable data bases, real-time access to mathematical models, and artificial intelligence systems will give rise to automated decision systems at command posts. The intent is not to create electronic generals, but rather to provide decision aids to commanders that can search through options and give either "optimal" solutions or a range of optimal variants.

The automated control of weapons systems (as opposed to troops) integrates weapons, sensors, and computers. Here man is removed from significant parts (or all) of the loop under normal operating circumstances. Automatic radar tracking systems, automated antiaircraft gun control, and automated missile control systems are examples most frequently cited in Soviet military writings. Mathematical models are used to devise control algorithms, to examine alternative control applications and operations, and to compare automated and non-automated operation modes. Fully automated weaponry is seen as being one approach that can lead to significant reductions in control times.

Thus, Soviet military work with mathematical modeling goes far beyond conventional analytic applications.

5.4 The Treatment of Risk and Uncertainty

Risk and uncertainty play important roles in the military policy of any state. Soviet military mathematical modeling explicitly treats two forms of uncertainty. The first is posed by the unknowable action of a responsive adversary. There is no definable data distribution. An adversary always has some range of choices available, and it is the uncertainty over the direction that he may choose that complicates planning. Gaming, simulation, and game theory serve as heuristic tools in discussions authored by Soviet military analysts.

Soviet specialists argue that when strategic level decisions are to be made, this form of uncertainty--and the risks it implies--are most effectively dealt with via a maximin approach. For the Soviet military this means that under a given Soviet choice of action the U.S. can be expected to do all that is possible to minimize Soviet gains. Therefore, Soviet planners should always assume a minimum outcome, regardless of other expectations. The maximin criterion says to select that course of action that offers the maximum of the minimal outcomes. This is a highly conservative, risk averse, strategy. It implies foregoing opportunities to strive for the maximum possible advantage. In the two instances most frequently noted, armaments selection and strategic planning, the maximin approach argues against an overt technology policy to achieve strategic breakout, but in favor of a covert technology policy to support breakout. In armaments selection, it also reinforces the bias against technological innovation, in favor of design and application innovation.

The second form of uncertainty pertains to random processes such as the weather. Here, stochastic and other statistical functions are used to simulate the statistical distribution of the phenomenon under study.

5.5 Mathematical Models and the Air Defense Missions

Many of our observations involving air defense modeling have already been noted. Nevertheless, it is worth considering them together. Moreover, since it is a mission area closely related to ballistic missile defense, and since both missions fall under the auspices of the same service--the Troops of Anti-air Defense, we may be able to develop some insights into Soviet military thinking regarding the programming of ballistic missile defenses.

The dynamics of the air defense mission are studied primarily with mass servicing models. Using such models, Soviet specialists can investigate "the struggle" between:

-- ECM and ECCM

-- stealth technology and new tracking technologies

-- offense tactics and defense tactics.

However, these models require that the many micro-level technical capablities of a given weapons system be collapsed into a few macro-level performance capabilities. These may be further truncated if the modeling algorithm handles heterogenous forces by converting components to equivalent weapon units. Consequently, the models tend to focus attention on what a given weapons system contributes to the air defense mission, and not the technical sources of those capabilities.

This tendency is reinforced by Soviet interest in using these models to assist in air defense staff work. To be useful, Soviet specialists argue that "staff models" must be workable under the constraints of time, space, computing power, and data likely to be encountered in command posts.

As has been noted previously, "time" is a particularly critical outcome variable in Soviet air defense modeling. This is amply illustrated in Soviet discussions of antiaircraft missile complexes (ZRKs) with small and large kill zones. Western specialists think of kill zones in terms of spatial envelopes. A weapon with a long slant range and a wide altitude regime would be seen as having a large kill zone. However, the Soviet notion of kill zone is rooted in service time. A ZRK has a small kill zone if its cycle time (the amount of time between engagement of different targets) is large compared to the amount of time the target is within the kill zone. A large kill zone denotes a small ZRK cycle time compared to target time in the "service queue."

Thus, control time and performance time determine the kill zone dimensions of a ZRK, not the spatial capabilities.

The effect of increasing the "waiting" time on the probability, P_{nf}, that an arriving aircraft is engaged by the air defense forces is shown in Figures 4 and 5. These results indicate that changing the waiting time does not have a decisive effect on P_{nf}, and consequently improvements in it would not significantly change the capabilities of Soviet air defenses. The only noticeable difference is that between the curves for $t_w = 0$ and those for $t_w > 0$, which serves to reinforce the Soviet distinction between ZRKs with small and large kill zones. The curve for $t_w = 10$ min. (a value which is probably unrealistically high) does not yield significant results. It is interesting to note that Soviet analysts went to considerable pains to include the effect of waiting time in their air defense models (compare the complexity of equations (13) and (14) above), even though the effect is not decisive.

Two basic technological directions are consistently adovcated: the increased use of automation technology and technologies to enhance antiair firepower density and maneuver. Automation technology is seen as the key to reducing control times in the command and control loop. Indeed, Soviet specialists argue that a breakout-like impact could be made by a significant advance in automated troop control. This might not be as observable as a new air defense missile, and therefore could be introduced without likelihood of detection. While the Korean airliner incidents of 1978 and 1983 suggest relatively poor Soviet command and control related intercept capabilities, two qualifiers are in order. First, the Soviet air defense system, as reflected by their modeling exercises, is not configured for optimal action against lone intruders.

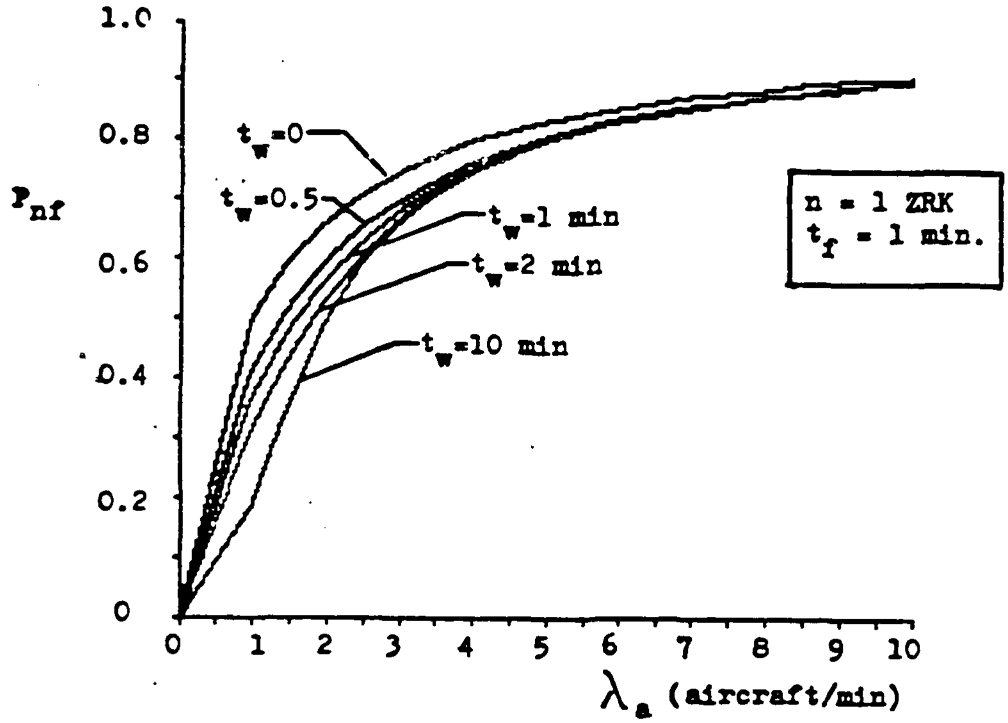


Figure 4. Effect of changing the "waiting" time on P_{nf}

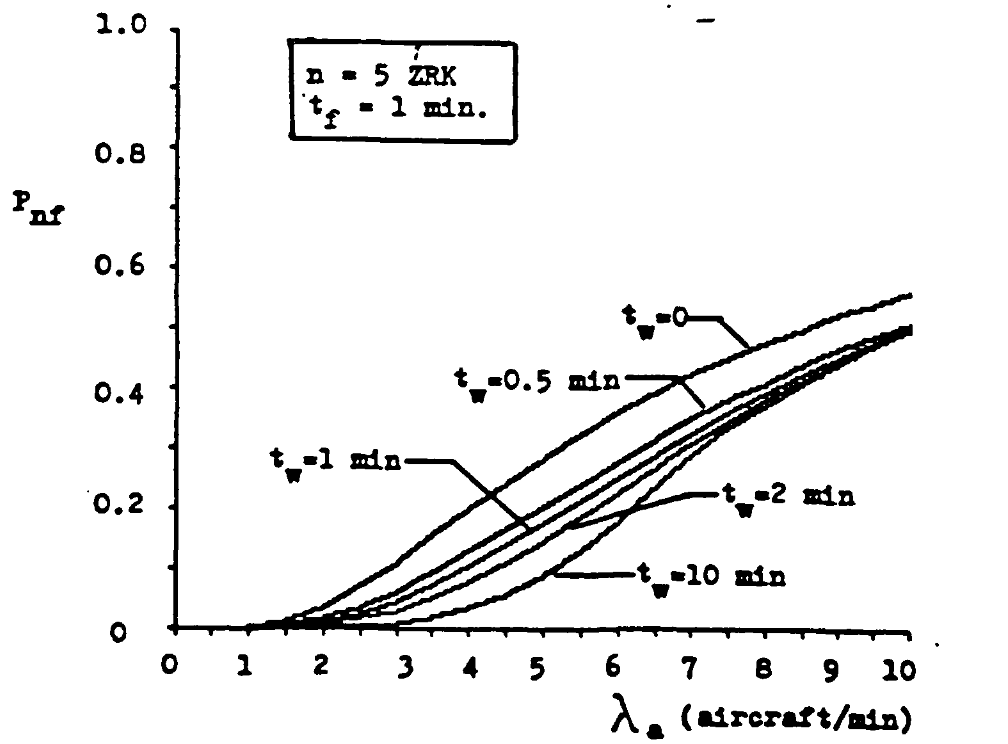


Figure 5. Effect of changing the "waiting" time on P_{nf}

Rather, it was designed to cope with massed air strikes. The requirements for detection, localization, command and control, and intercept are quite different. Second, both incidents involved Soviet interceptor aviation, not the Zenith Rocket Troops (antiaircraft missile forces). The latter are portrayed as being the core of Soviet air defense capabilities and are also the primary focus of automated control technology work in the Soviet Union.

Unlike "waiting" time, "service" time (or the total time it takes a ZRK to shoot at an attacking aircraft) has a dramatic effect on P_{nf} (see Figures 6 through 9). As is shown in the appendix, the firing cycle of an SA-3 is approximately 1 min. and an FB-111 remains within its kill zone for approximately 45 sec. (a figure which was used in constructing Figures 7 and 9). For example, Figure 9 shows that if the service time were decreased from 1 min. to 30 sec. then P_{nf} is decreased from approximately 50% (or one half of the attacking aircraft penetrate without being fired upon) to 15% (or nearly all enemy aircraft are at least fired upon, if not destroyed). This effect is much more pronounced than that noted above for similar changes in waiting time. These results effectively highlight Soviet interest in minimizing the length of SAM firing cycles. Minimizing the control time through the use of automation could, therefore, greatly improve the performance of Soviet air defense forces. In addition the Soviet military gives considerable attention to increasing the "combat readiness" and efficiency of PVO troops -- the effect of which is to minimize the time required to prepare and fire air defense missiles.

Another important consideration which is reflected in Soviet air defense models is the degree to which the individual ZRKs are coordinated

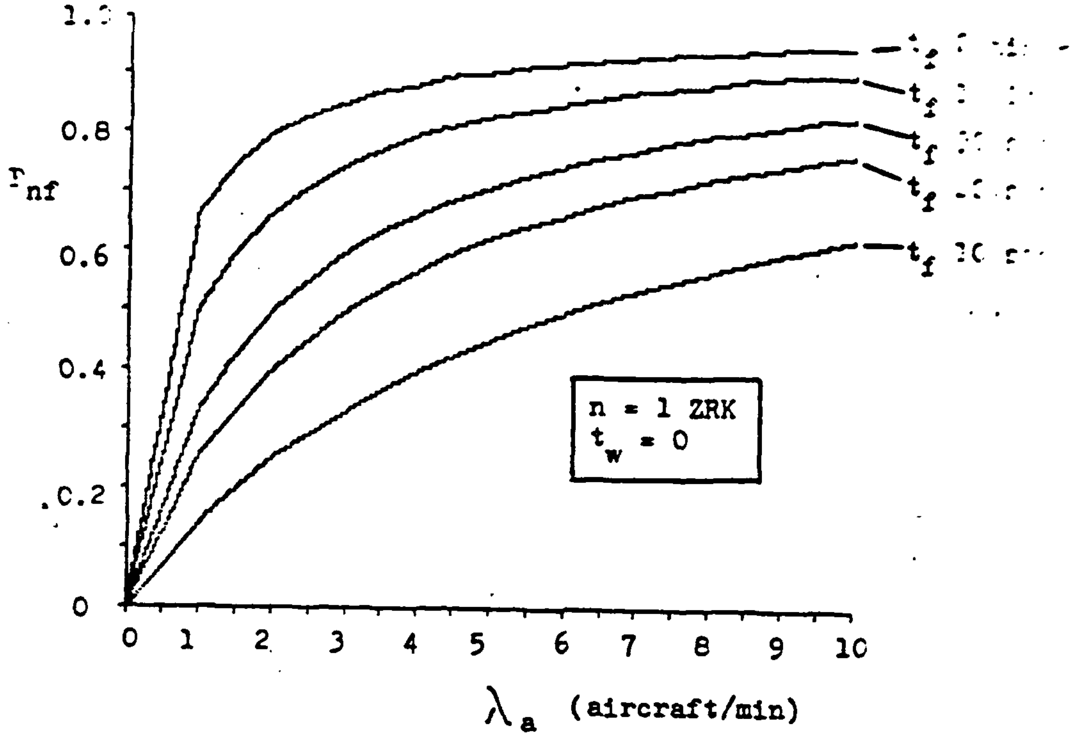


Figure 6. Effect of changing the "service" time on P_{nf}

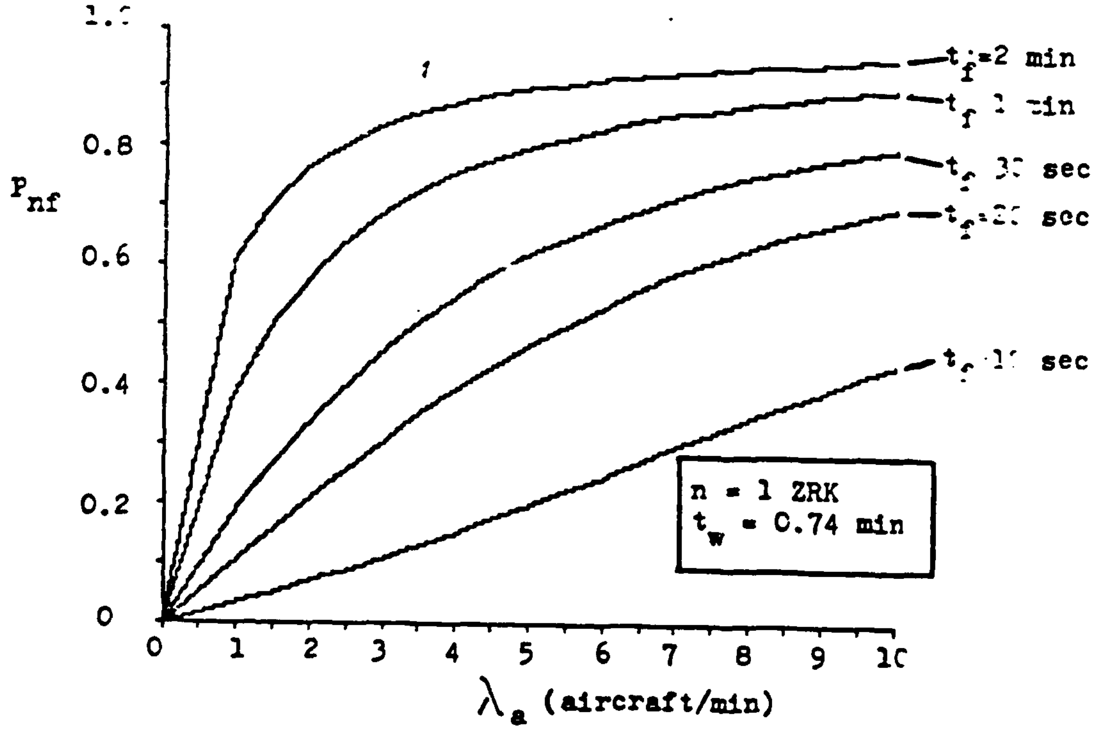


Figure 7. Effect of changing the "service" time on P_{nf}

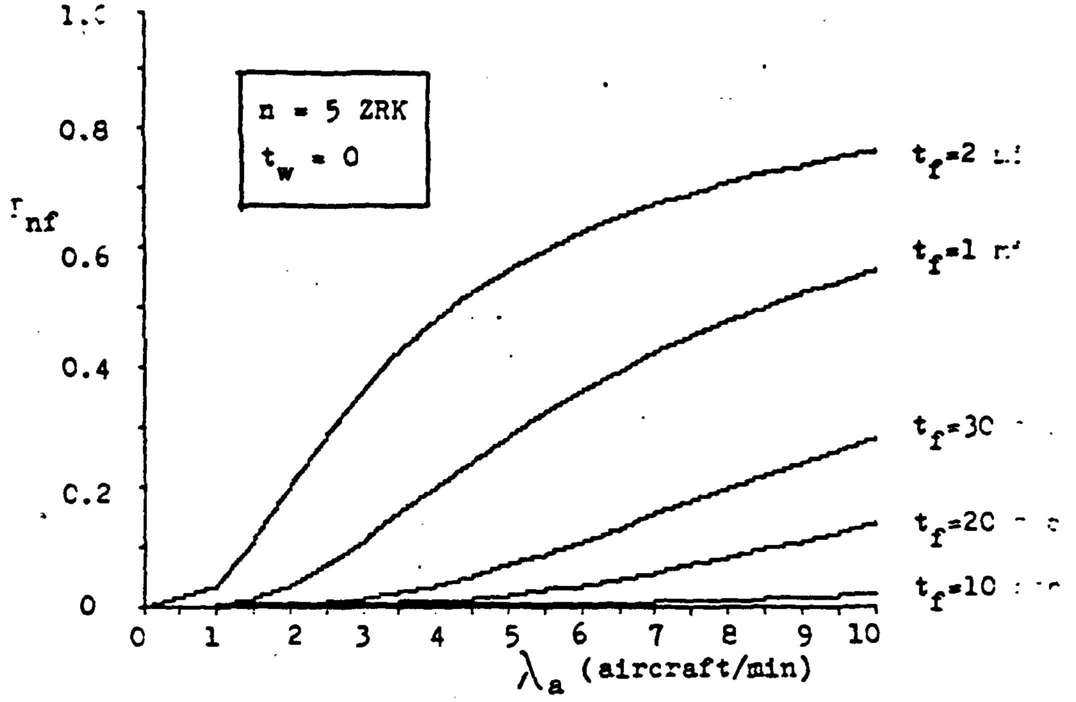


Figure 8. Effect of changing the "service" time on P_{nf}

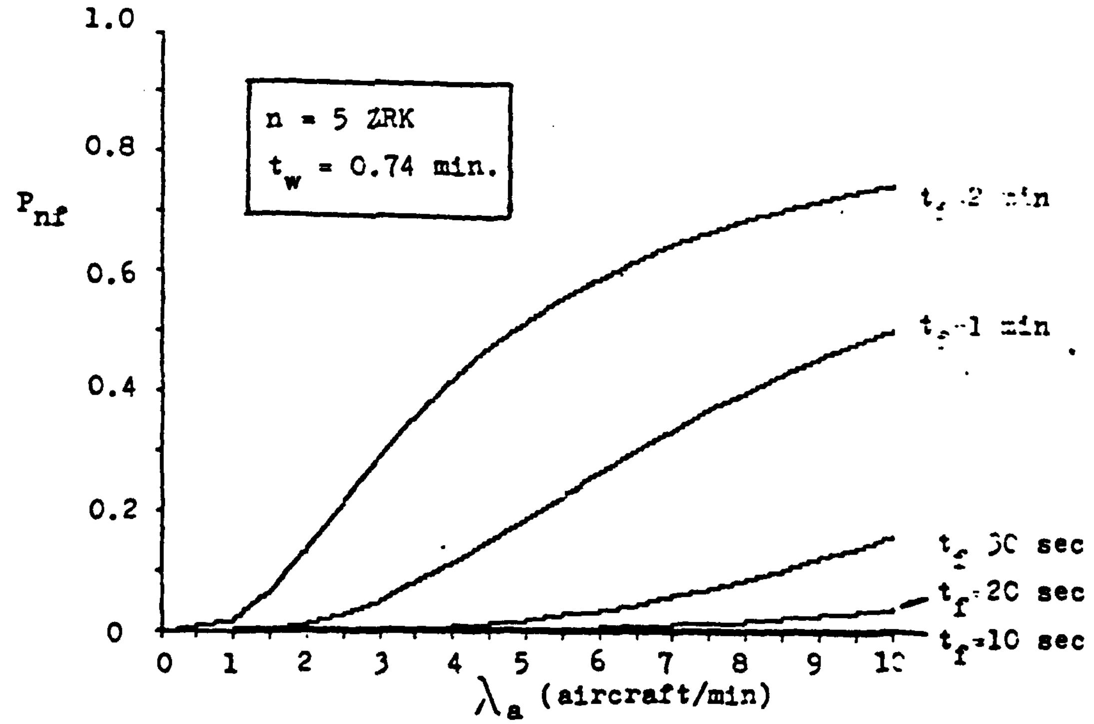


Figure 9. Effect of changing the "service" time on P_{nf}

by a central command post. In the limit of perfectly "controlled" forces an arriving aircraft is engaged by one and only one of the available ZRKs. In contrast, with "uncontrolled" forces an arriving aircraft is engaged by all available ZRKs since there is no central command post to designate which of the available ZRKs is to fire on which enemy aircraft. This distinction is actually based on two different firing doctrines, one of which requires a central command post and the other which does not. This model (described in the appendix) indicates that except for a few cases central control, or lack thereof, does not exert a decisive influence. Figures 10 through 13 show some results based on this model, where W is the probability that an arriving aircraft is destroyed and R is the kill probability for a single ZRK. For large values of R (or for highly effective SAMs) centralized control is important, but otherwise its overall effect appears to be small. For small values of λ_a, "uncontrolled" forces appear to be better than "controlled" ones, but this apparent contradiction is an artifact of the different firing doctrines which lie behind the two models: it is better to concentrate fire on each aircraft (i.e., to leave the forces "uncontrolled") if the average interarrival time is long compared to the firing cycle. Soviet concerns regarding the vulnerability of their centralized command and control capabilities may be reflected in this modeling work.

Density of antiair activity and "fire maneuver" capabilities (the ability to rapidly shift the direction of fire) seem to overshadow individual weapon lethality in contemporary Soviet modeling work. That is to say, as a mass servicing problem, qualitative improvements that do not directly affect the cycle time of individual weapons do not

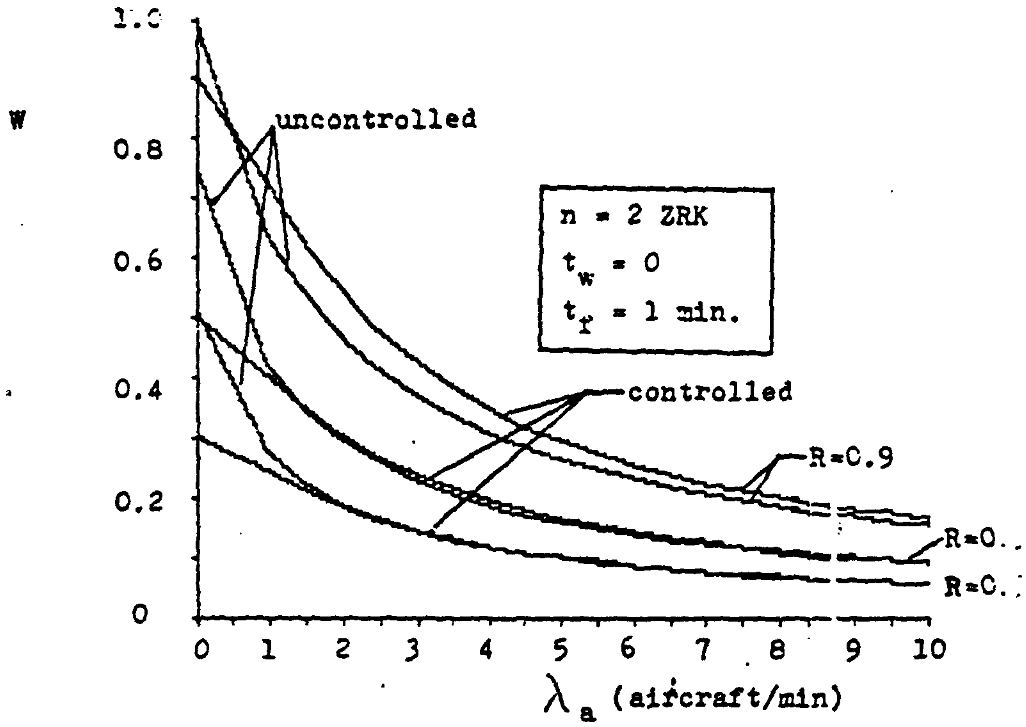


Figure 10. Effect of "controlled" vs. "uncontrolled" forc[illegible] on W

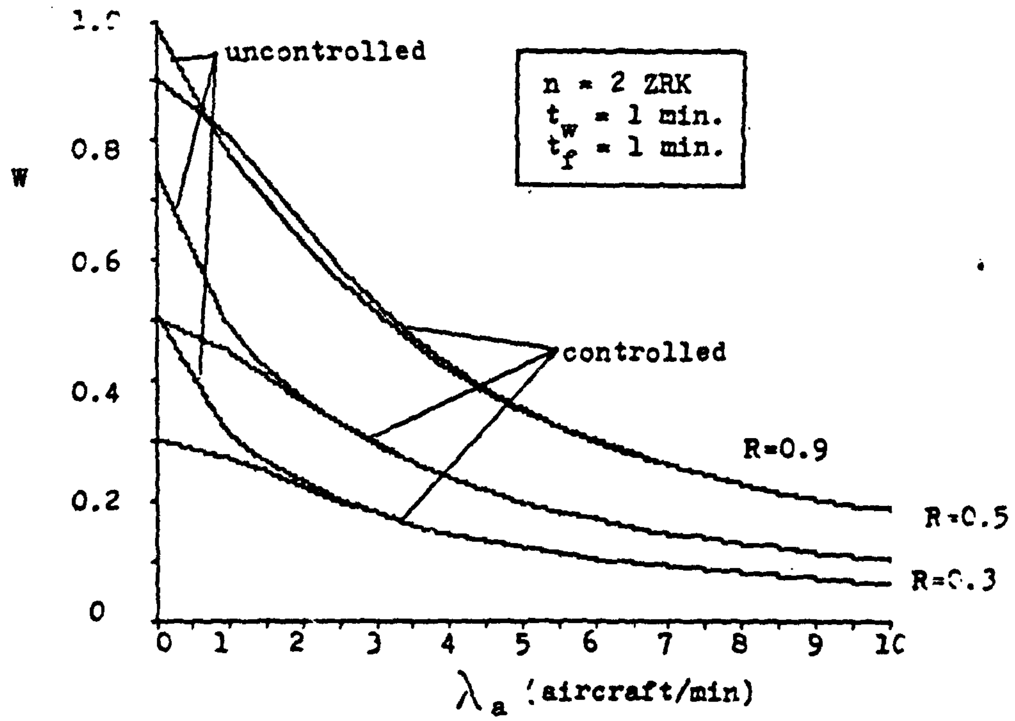


Figure 11. Effect of "controlled" vs. "uncontrolled" forces on W

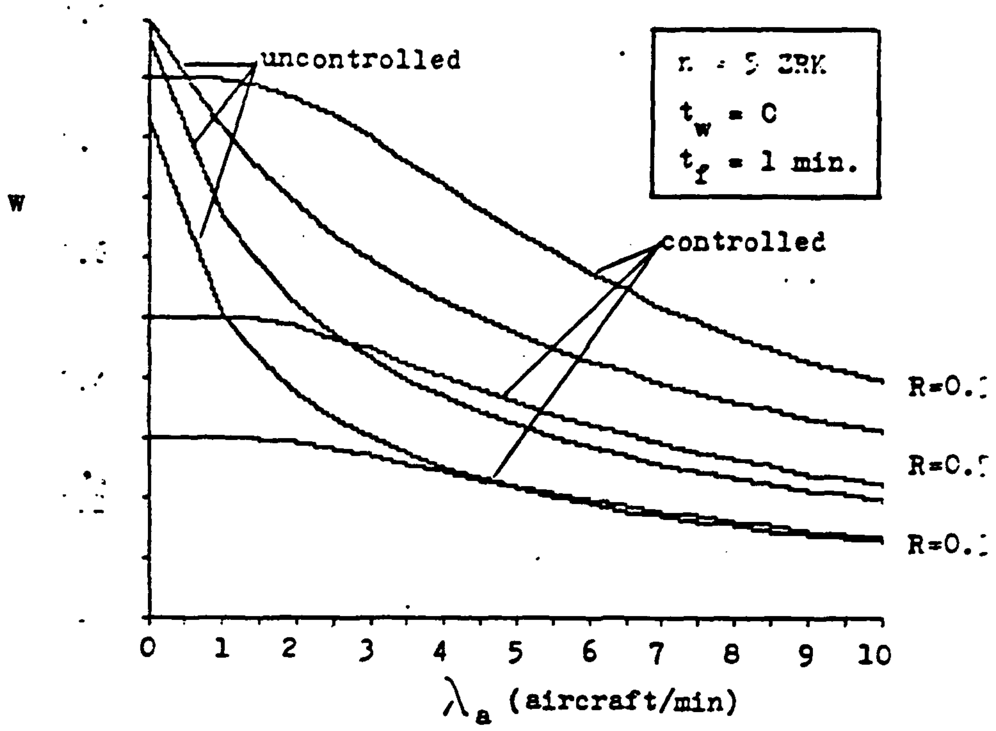


Figure 12. Effect of "controlled" vs. "uncontrolled" forces on W

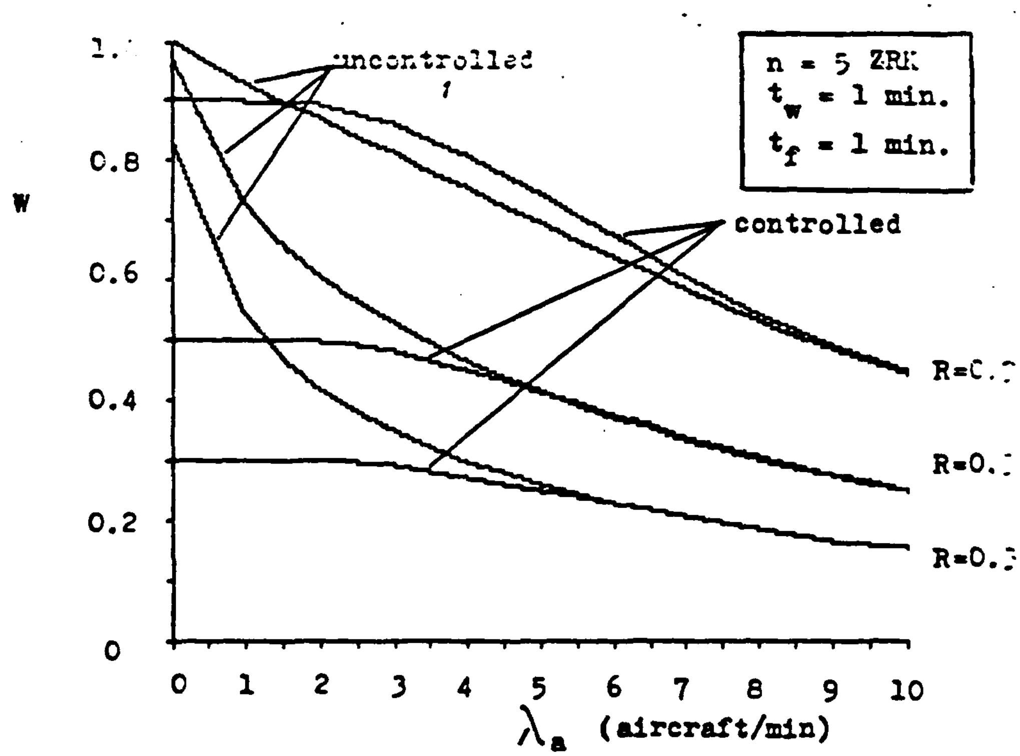


Figure 13. Effect of "controlled" vs. "uncontrolled" forces on W

contribute as greatly towards increasing the rate of "servicing" as do quantitative improvements. Again, the problem is structured as a struggle against a quantitatively large threat (mass servicing), and therefore is seen as a "many against many" mission, not one on one.

Soviet military mathematical modeling in general, and modeling for air defense purposes in particular, distinguish between the combat capabilities of weapons and the combat effectiveness of weapons. The former is a property of the hardware, while the latter is a broader quality that includes the impact of human operators, enemy responses and countermeasures, command and control, etc. This conception of weapon effectiveness also accents the role of time in evaluating a system's relative utility.

Taken together, the above mentioned factors tend to support the observed Soviet proclivity to retain old equipment in their active inventory, and to innovate incrementally when precursor systems are already deployed in large quantities. Design innovation and application innovation dominate technological innovation where existing weaponry are concerned. This does not preclude, however, the introduction of new technological approaches to accomplish existing missions. For example, the long range missile was rapidly accepted in place of aircraft for many strategic missions. Likewise, a laser antiaircraft system would be readily brought into the inventory to replace antiaircraft missiles. However, the Soviet air defense analyses would rate the initial military impact of any such technological innovation as quite small until it could be deployed in quantities that would significantly alter the mass servicing rate. (This does not consider the military-psychological impact that an initial deployment might have.) Thus, military breakout would require clandestine serial production to the point where substantial numbers of operational weapons would be available. It took the U.S.S.R. more than a decade to convert their novel ICBM into a military system that eclipsed the existing capabilities of their long-range aviation force.

The use of norms in Soviet air defense modeling is rooted in two realizations. First, to date no one has been able to devise a leak proof air defense. Moreover, beyond some point marginal improvements in air defense capability are disproportionately costly. Second, Soviet historical studies suggest that at a certian level of attrition, far below 50%, enemy air attacks are called off. Thus, an effective and efficient air barrier can be established even though leakage rates are very high, assuming that all sorties can be deterred by a formidable defense. Of course there are two caveats. One: this strategy may not work in a strategic nuclear war in which aircraft would be expected to make only a single sortie--that is, 60% to 75% attrition rates might not deter. Yet, it is possible that Soviet military planners reason that they can deflect strategic bombing missions away from high-value targets, redirecting them to lower value targets, through such a deterrent concept. Two: more significantly, cruise missile delivery cannot be deterred in this manner. Indeed, our examination of this modeling material suggests that the advent of the strategic cruise missiles undermines one of the fundamental assumptions of Soviet strategic air defense planning in the post-war period.

Soviet interest in the modeling of combat readiness suggests something about their concern about the time-sensitivity of the effectiveness of their air defense system. How long does it take to surge the air defense network from its relaxed peacetime state to its maximum alert state? How long can the air defense network stay a maximum alert before readiness decays significantly? Here, Soviet specialists prefer to work with statistical data. Frequent Soviet air defense exercises supply plenty of data for such studies and for the generation of "norms." These exercises, however, for the most part involve division-sized formations or lower. Lacking are theater or nation level exercises that would be necessary to provide a systemic picture.

Footnotes

1. Moskvin, et al (1964; 60).
2. Anureyev, et al (1963; 20).
3. Anureyev, et al (1963; 33).
4. Pevnitskiy (1964; 11).
5. Pevnitskiy (1964; 9).
6. Pevnitskiy (1964; 10-11).
7. Pevnitskiy (1964; 13).
8. Pevnitskiy (1964; 12).
9. Bazanov and Malinovskiy (1964; 16).
10. Moskvin, et al (1964; 55).
11. Moskvin, et al (1964; 55-56).
12. Moskvin, et al (1964; 56).
13. Moskvin, et al (1964; 56-57).
14. Ryabchuk (1971; 111).
15. Moskvin, et al (1964; 60).
16. Anureyev, et al (1963; 23).
17. Anureyev, et al (1963; 20).
18. Bazanov and Malinovskiy (1964; 15).
19. Tarakanov and Tsygichko (1972; 69).
20. Lomov (1973; 238-245).
21. Volkov (1975; 22).
22. Dmitriyev (1965; 10).
23. Tatarchenko (1976; 1).
24. Anureyev, et al (1963; 24).
25. Dmitriyev (1965; 10-11).

26. Anureyev (1966; 47).

27. Tarakanov and Tsygichko (1972; 70).

28. Tarakanov and Tsygichko (1972; 76).

29. Dmitriyev (1965; 8-9).

30. Tatarchenko (1976; 2).

31. Anureyev, et al (1963; 31).

32. Tarakanov and Tsygichko (1972; 76).

33. Tatarchenko (1976; 5).

34. Moskvin, et al (1964; 57).

35. Pul'kin (1971; 85).

36. Anureyev (1966; 50).

37. Neupokoyev (1973; 87-90).

38. Anureyev, et al (1963; 33-34).

39. Pevnitskiy (1964; 13).

40. Moskvin, et al (1964; 60).

41. Ryabchuk (1971; 111).

42. Gusev (1963; 37).

43. Gusev (1963; 32).

44. Ryabchuk (1971; 111).

45. Tatarchenko (1976; 3).

46. Tatarchenko (1976; 3).

47. Anureyev (1972; 33).

48. Anureyev, et al (1963; 33). Emphasis added.

49. Tatarchenko (1966; 58). Emphasis added.

50. Moskvin, et al (1964; 59).

51. Bazanov and Malinovskiy (1964; 17).

52. Volkov (1975; 27).

53. Ionov and Beglaryan (1974; 8-9).

54. Ryabchuk (1971; 117-118).

55. Tatarchenko (1976; 5).

56. Anureyev (1966; 53).

57. Lomov (1973; 238).

58. Anureyev, et al (1963; 20).

59. Tatarchenko (1976; 8).

60. Tatarchenko (1976; 3).

61. Anureyev, et al (1963; 25).

62. Anureyev, et al (1963; 26).

63. Bazanov and Malinovskiy (1964; 18-19).

64. Anureyev, et al (1963; 26).

65. Anureyev, et al (1963; 22).

66. Anureyev, et al (1963; 22).

67. Anureyev, et al (1963; 27).

68. Anureyev, et al (1963; 27).

69. Moskvin, et al (1964; 60).

70. Pul'kin (1972; 87).

71. Pul'kin (1972; 87).

72. Pul'kin (1972; 90).

73. Bazanov and Malinovskiy (1964; 18).

74. Anureyev, et al (1963; 30).

75. Anureyev, et al (1963; 30).

76. Anureyev, et al (1963; 29).

77. Anureyev, et al (1963; 30-31).

78. Bazanov and Malinovskiy (1964; 18).

79. Tarakanov and Tsygichko (1972; 75).

80. Anureyev (1966; 55).
81. Anureyev (1966; 56).
82. Volkov (1975; 25).
83. Volkov (1975; 26).
84. Anureyev (1966; 58).
85. Volkov (1975; 26).
86. Anureyev (1966; 58).
87. Romanov and Frolov (1971; 318-326).
88. Anureyev (1966; 58).
89. Anureyev (1966; 59).
90. Volkov (1975; 26).
91. Anureyev (1966; 52).
92. Anureyev (1966; 44-54).
93. Dmitriyev (1965; 13).
94. Dmitriyev (1965; 13-14).
95. Dmitriyev (1965; 14).
96. Moskvin, et al (1964; 60).
97. Kholodov and Sidorenko (1971; 34).
98. Solnyshkov (1973b; 43).
99. Solnyshkov (1973b; 44).
100. Solnyshkov (1973b; 47).
101. Tatarchenko (1966; 76-77).
102. Tatarchenko (1966; 63).
103. Anureyev (1966; 55).
104. Solnyshkov (1973b; 54).
105. Solnyshkov (1973b; 54).
106. Solnyshkov (1973b; 55).

107. Solnyshkov (1973b; 55).

108. Solnyshkov (1970; 29).

109. Solnyshkov (1973b; 56-57).

110. Tsybul'ko (1972; 42).

111. Tsybul'ko (1972; 42).

112. Tsybul'ko (1972; 43).

113. Tsybul'ko (1972; 44).

114. Tsybul'ko (1972; 46).

115. Solnyshkov (1968; 96-97).

116. Tsybul'ko (1972; 46).

117. Tsybul'ko (1972; 47).

118. Tsybul'ko (1972; 47).

119. Tsybul'ko (1972; 47-48).

120. Tsybul'ko (1972; 49).

121. Anureyev (1967; 37).

122. Lozik and Petukhov (1973; 77).

123. Lozik and Petukhov (1973; 77).

124. Lozik and Petukhov (1973; 77-78).

125. Lozik and Petukhov (1973; 76).

126. Lozik and Petukhov (1973; 78).

127. Lozik and Petukhov (1973; 82).

128. Lozik and Petukhov (1973; 86).

129. Lozik and Petukhov (1973; 83).

130. Lozik and Petukhov (1973; 84).

131. Lozik and Petukhov (1973; 84-85).

132. Lozik and Petukhov (1973; 85).

133. Botin and Ivankov (1973; 88).

134. Botin and Ivankov (1973; 88).

135. Botin and Ivankov (1973; 89).

136. Botin and Ivankov (1973; 89).

137. Botin and Ivankov (1973; 90).

138. Botin and Ivankov (1973; 90).

139. Botin and Ivankov (1973; 90).

140. Botin and Ivankov (1973; 91).

141. Botin and Ivankov (1973; 91).

142. Botin and Ivankov (9173; 92).

143. Botin and Ivankov (1973; 93).

144. Botin and Ivankov (1973; 94).

145. Tarakanov and Tsygichko (1972; 68).

146. Sosura, et al (1964; 57).

147. Gusev (1963; 41).

148. Lozik and Petukhov (1973; 74).

149. Anureyev (1972; 33).

150. Anureyev (1967; 61).

151. Solnyshkov (1973a; 120).

152. Solnyshkov (1973a; 122).

153. Ryabchuk (1971; 114).

154. Gusev (1963; 41).

155. Anureyev (1966; 55).

156. Anureyev (1967; 43-44).

157. Semeyko (1968; 55).

158. Semeyko (1968; 56).

159. Anureyev (1972; 38).

160. Anureyev (1972; 33).

161. Anureyev (1972; 38).

162. Anureyev (1972; 36).

163. Berman and Baker (1982; 87-89).

164. Smirnov and Bazanov (1968; 100).

165. Smirnov and Bazanov (1968; 101).

166. Smirnov and Bazanov (1968; 103).

167. Ryabchuk (1971; 118).

168. Smirnov and Bazanov (1968; 104).

169. Anureyev, et al (1967; 29).

170. Anureyev (1972; 38).

171. Anureyev (1967; 38-39).

172. Tatarchenko (1976; 11).

173. Anureyev (1966; 54).

174. Anureyev (1966; 55).

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Appendix

1. Variables used in analysis

λ_a = arrival rate of attacking aircraft, generally expressed in aircraft per minute.

t_f = mean service time for an antiaircraft complex (ZRK).

ν = service rate, aircraft per minute,

= $1/t_f$.

n = number of independent air defense channels, generally equal to the number of ZRKs.

t_w = mean waiting time for aircraft, or the average time that aircraft spend in a ZRK's kill zone.

μ = "waiting rate,"

= $1/t_w$.

$\alpha = \lambda_a / \nu$.[1]

$\beta = \lambda_a / \mu$.

E_k = a state in which there are k aircraft in the system; note that for $k > n$, n aircraft are being "serviced" and k-n are "waiting for service."

$P_k(t)$ = probability that the system in state E_K at time t.

P_K = equilibrium value of $P_K(t)$.

$[x]$ = greatest integer less that or equal to x.

2. Antiaircraft Rocket Complex (ZRK) with small kill zone

Soviet analysts model air defense engagements using queuing theory apparently because they are concerned about air defense systems becoming "saturated" as attacking aircraft arrive at a rate faster than they can be engaged. Queuing models place heavy emphasis on time constraints, and less on resource constraints (i.e., the limited number of antiaircraft guided missiles, ZURs, available to engage the attacking aicraft). An important criterion of efficiency for ZRKs is the probability that an arriving aircraft will be engaged by the air defense forces:

$$P_f = 1 - P_{nf}, \qquad (1)$$

where P_{nf} is the quantity that is directly calculated.

The following set of differential equations represent the queuing system used to model ZRKs with small kill zones[1]:

$$\frac{dP_o}{dt}(t) = -\lambda_a P_o(t) + \nu P_1(t), \qquad (2)$$

$$\frac{dP_k}{dt}(t) = -(\lambda_a + k\nu)\ P_k(t) + \lambda_a P_{K-1}(t) + (k+1)\nu P_{K+1}(t), \qquad (3)$$

$$\frac{dP_n}{dt}(t) = -n\nu P\ (t) + \lambda_a P_{n-1}(t). \qquad (4)$$

These equations can be interpreted by noting that the system can leave state E_K in one of two ways: (1) a plane arrives, causing a transition to state E_{K+1}; or (2) one of the k planes being serviced finishes, causing a transition to state E_{K-1}. The system can also get into state E_K in one of two ways: (1) the system was in state E_{K-1} and a plane arrives; or (2) it was previously in state E_{K+1} and one of the k+1 planes finished service. If higher order transitions are not allowed (e.g., two planes never arrive exactly simultaneously), then these transitions define the terms in equation (3). Equations (2) and (4) differ only in that they are at the boundaries of the available states. P_0 (t) can change only if a plane arrives while the system is in E_0 or one finishes service while the system is in E_1 ; and similarly P_n (t) can change only if the system is in E_n and one plane finishes service or if it was in E_{n-1} and a plane arrives. Note that $P_k(t) = 0$ for $k > n$ since it is assumed that planes cannot be serviced if no units are available when the plane arrives, or there is no waiting. Physically this condition means that the ZRK's kill zone (limited by the range of the ZUR or the radar's detection range at the aircraft's altitude) is sufficiently small that the plane remains within range for too short a period of time for it to be engaged unless a ZRK is immediately available.

Generally only the equilibrium solution for equations (2)-(4) is determined, where $P_k = \lim_{t \to \infty} P_k(t)$. By setting the time derivatives in equations (2)-(4) equal to zero, the following is obtained:

$$0 = -\lambda_a P_0 + \nu P_1 , \quad (5)$$
$$0 = -(\lambda_a + k\nu) P_k + \lambda_a P_{k-1} + (k+1)\nu P_{k+1} , \quad (6)$$
$$0 = -n\nu P_n + \lambda_a P_{n-1} . \quad (7)$$

Lozik and Petukhov noted that investigations indicated that the steady-state solutions are valid approximations if the attack duration exceeds two to three times the service time[2] or:

$$T_a > 3 t_f .$$

Since the system must be in some state, an additional constraint is:

$$\sum_{K=0}^{n} P_K = 1; \tag{8}$$

noting again that $P_K = 0$ for $K > n$.

These equations can be solved iteratively by expressing all P_k in terms of P_o and then using equation (8) to determine P_o. Equation (5) yields:

$$P_1 = \left(\frac{\lambda_a}{\nu}\right) P_o$$

$$= \alpha P_1 .$$

Using k=1 in equation (6), one obtains the following:

$$0 = -(\lambda_a + \nu) P_1 + \lambda_a P_o + 2 \nu P_2 ,$$

or,

$$P = 1/2 (\lambda_a / \nu)^2 P_o ,$$
$$= (\alpha^2 /2!) P_o , \tag{10}$$

Repeating this procedure with k=3 yields:

$$P_3 = (\alpha^3 /3!) P_o . \tag{11}$$

These results imply that for $1 \leq k \leq n$:

$$P_K = (\alpha^k /k!) P_o , \tag{12}$$

which can be verified by insuring that this formula implies a similar relationship for P_{K+1}, or that:

$$P_{K+1} = (\alpha^{k+1}/(k+1)!)\, P_o \quad .$$

Rewriting equation (6):

$$P_{K+1} = \frac{1}{(k+1)\nu}\left((\lambda_a + k\,\nu)\, P_K - \lambda_a P_{k-1} \right)$$

$$= \frac{1}{(k+1)}\left((\alpha + k)\ (\alpha^k/k!) - (\alpha^{k-1}/(k-1)!) \right) P_o$$

$$= (\alpha^{k+1}/(k+1)!)\, P_o \quad .$$

The value of P_o can be then found by solving equation (8):

$$P_o = 1/\left(\sum_{k=0}^{n} \alpha^k/k! \right) \tag{14}$$

Note that the probability that an aircraft will pass through the air defense forces unattacked is equal to the probability that all n channels are occupied, or;

$$P_{nf} = P_n = \frac{\alpha^n/n!}{\sum_{k=0}^{n} \frac{\alpha^k}{k!}}, \tag{15}$$

which is equation (13) in section 3.2 above.

Figure A-1 shows how P_{nf} varies with λ_a for various values of n. For $\alpha = 10$, which corresponds to $\lambda_a = 10$ aircraft per minute and $t_f = 1.0$ minute (the approximate length of the SA-3 firing cycle)[3], then n must be greater than 10 for the air defense forces to have at least an 80% chance of firing at (not destroying) an aerial target.

Figure A-2 shows the kill zones for several Soviet antiaircraft missiles: SA-2, SA-3, and SA-6. The SA-2, for which there is a reasonable amount of data available pubicly, is an example of a ZRK with a small kill zone. The firing cycle is between 10 and 12 min and its fire control radar, Fan Song, can track 6 targets simultaneously but guide missiles to only one.[4] Thus an SA-2 ZRK can engage only one target every 10 to 12 min. In a Soviet textbook on the fire control of antiaircraft missiles, A.S. Mal'gin cited maximum speeds for some U.S. bombers; for example the maximum speed for a B-52 at 5 km is given as 1200 km/hr and that for a B-1 as 2330 km/hr.[5] Using 5-10 km as typical values for high altitude penetration, the ranges to the near and far boundaries of the SA-2's kill zone are approximately:

at 5 km

$$D_r = 43.5 \text{ km},$$
$$d_r = 10.0 \text{ km},$$

at 10 km

$$D_r = 53 \text{ km},$$
$$d_r = 6 \text{ km}.$$

Thus for a B-52 flying at V_t = 1200 km/hr at an altitude of 5 km would remain within range of an SA-2 ZRK for:

$$t_w = (43.5\text{-}10)\ \text{km} /(1200\ \text{km/hr})$$
$$= 1.7\ \text{min.}$$

Since the firing cycle is 10 minutes long, unless the SA-2 ZRK is available when a B-52 arrives, it is unlikely that the plane would be fired upon. At an altitude of 10 km, t_w =2.4 min which is still much shorter than t_f . The comparable figures for a B-1 are 0.86 and 1.2 min, respectively. Thus despite the apparent long range, the SA-2 can be represented as a ZRK with a small kill zone.

3. ZRK with large kill zone

If the ZRK's kill zone is sufficiently large then arriving aircraft do not necessarily pass through the system unattacked if all ZRKs are "busy" at the moment the aircraft arrives. In other words, the planes "wait" for service if it is not immediately available. A possible motivating factor behind this Soviet model of air defense was a set of two articles published by Donald Barrer (from the Institute for Defense Analysis) in Operations Research in 1957. In these articles Barrer considered queuing processes in which customers wait for only a limited time. Among the possible areas of application for this analysis mentioned by Barrer was air defense:

> Many types of military engagements are similarly characterized (by limited waiting). An attacking airplane engaged by anti-aircraft or guided missiles is available for "service," i.e., is within range, for only a limited time. It is of interest to relate the

> expected rate at which aircraft are shot down to the firing rate and accuracy of the defensive weapons, the rate at which aircraft come within range, and the time that each airplane would remain within range if not shot down.[6]

Although the journal Operations Research has published several articles describing air defense models, only the Barrer articles mentioned the potential use of queuing theory to model air defense engagements. Nevertheless, in a standard textbook on queuing theory, B. Gnedenko and I. Kovalenko cited the above passage in the Barrer article and noted that this general class of problems was of great importance in military affairs. They further noted that the specific problem solved by Barrer, where the waiting time was assumed to be constant, was of less practical interest than one where the waiting is assumed to be a random variable.[7]

Much of the following derivation of the Venttsel' formula is based on the Gnedenko-Kovalenko book cited above. However they neither explicitly derive the Venttsel' formula nor directly cite any work by Ye. S. Venttsel'. A set of differential equations, similar to those used above, describe this queuing process, except that the transition rates are more complicated. For a general state E_k :

$$\frac{dP_k}{dt}(t) = -(\lambda_k + \nu_k)\, P_k(t) + \lambda_{k-1} P_{k-1}(t) + \nu_{k+1} P_{k+1}(t), \quad (16)$$

where λ_k and ν_k are the rates at which "customers" arrive and leave, respectively, if the system is in state E_k. The value of λ_k is always λ_a, or the arrival rate of attacking aircraft, regardless of the state of the system. For $k \leq n$, $\nu_k = k\,\nu$, as above, since any one of the k aircraft being "served" can finish; note $\nu_0 = 0$. However for

$k > n$ ν_k must account for the waiting process. Gnedenko and Kovalenko assume that the waiting time is limited by a random variable with an exponential distribution[7]:

$$Pr(t_w < \tau) = 1 - \exp(-\rho \tau), \qquad (17)$$

where ρ is a constant determined by the average waiting time. Waiting is thus a simple Markov process, analogous to service. For $k > n$,

$$\nu_k = n\nu + (K-n)\mu , \qquad (18)$$

or any one of the n planes being serviced can finish service or any one of the k-n planes waiting can leave without being serviced. The assumption that the process of planes waiting can be represented by a Markov process simplifies the mathematics but is probably not physically accurate. The probability that a plane leaves is not independent of its past history (an important assumption in a Markov process). Lozik and Petukhov did not mention this limitation.

The following differential equations can be used to represent this queuing process:

$$\frac{dP_o}{dt}(t) = -\lambda_a P_o(t) + \nu P_1(t); \qquad (19)$$

for $1 \leq k \leq n$:

$$\frac{dP_k}{dt}(t) = -(\lambda_a + k\nu) P_k(t) + \lambda_a P_{k-1}(t) + (k+1)\nu P_{k+1}(t); \qquad (20)$$

for k=n:

$$\frac{dP_n}{dt}(t) = -(\lambda_a + n\nu)\ P_n(t) + \lambda_a P_{n-1}(t) + (n\nu + \mu)\ P_{n+1}(t); \qquad (21)$$

for k>n:

$$\frac{dP_K}{dt}(t) = -(\lambda_a + n\nu + (k-n)\mu)\ P_K(t) + \lambda_a P_{K-1}(t) + (n\nu + (k-n+1)\mu)\ P_{K+1}(t). \qquad (22)$$

Unlike the queuing model for ZRKs with small kill zones, it is not immediately obvious that an equilibrium solution exists. The general condition for the existence of a stable equilibrium solution, given by Gnedenko and Kovalenko, is that for some k the following inequality must hold[9]

$$\lambda_k / \nu_{k+1} < 1, \qquad (23)$$

where λ_k is the rate at which the queuing system makes the transition from state E_K to E_{K+1} and ν_{K+1} is the rate for the reverse transition from E_{K+1} to E_K. This condition requires that at some point downward transitions occur at a faster rate than upward ones; thus insuring that the length of the queue does not grow indefinitely. The forms for λ_K and ν_K given above insure that this condition is satisfied as long as $\mu \neq 0$. For k>n this inequality becomes:

$$\lambda_a/(n\nu + (k-n+1)\mu) < 1,$$

or,

$$k > k_o = [(\lambda_a - n\nu)/\mu + n]. \qquad (24)$$

Note that k_o is a well defined integer so long as $\mu \neq 0$, or that t_w is finite.

The equilibrium solution to equations (19)-(22) can be found iteratively. For $0 \leq k \leq n$ these equations are identical to those for ZRKs with small kill zones and thus have the same solutions:

$$P_k = (\alpha^k / k!)\, P_o \quad . \qquad (25)$$

For $K=n$:

$$0 = -(\lambda_a + n\nu)\, P_n + \lambda_a P_{n-1} + (n\nu + \mu) P_{n+1} \quad ,$$

or

$$P_{n+1} = \frac{1}{n!} \; \frac{\lambda_a^{n+1}}{\nu^n} \; \frac{1}{(n\nu + \mu)} \; P_o \; . \qquad (26)$$

Using equation (22) with k=n+1:

$$0 = -(\lambda + n\nu + \mu)\, P_{n+1} + \lambda_a P_n + (n\nu + 2\mu) P_{n+2} ,$$

or

$$P_{n+2} = \frac{1}{n!} \; \frac{\lambda_a^{n+2}}{\nu^n} \; \frac{1}{(\nu + \mu)(\nu + 2\mu)} \; P_o \; . \qquad (27)$$

Equations (26) and (27) imply that in general:

$$P_K = \frac{1}{n!}\frac{\lambda_a^k}{\nu^n}\frac{1}{\prod_{m=1}^{k-n}(n\nu + m\mu)} P_o , \qquad (28)$$

which can be verified by insuring that this equation implies a similar relation for P_{K+1}. Using equation (22) and substituting values for P_K and P_{K-1} using equation (28):

$$(n\nu + (k-n+1)\mu)\ P_{K+1} = P_o\left((\lambda_a + n\nu + (k-n)\mu)\frac{\lambda_a^k}{n!\,\nu^K \prod_{m=1}^{k-n}(n\nu + m\mu)} - \lambda_a \frac{\lambda_a^{k-1}}{n!\,\nu^n \prod_{m=1}^{k-n-1}(n\nu + m\mu)}\right)$$

which can be simplified to show that:

$$P_{K+1} = \frac{\lambda_a^{K+1}}{n!\ \nu^n}\frac{1}{\prod_{m=1}^{K-n+1}(n\nu + m\mu)}.$$

Finally the value for P is obtained by requiring:

$$\sum_{K=0}^{\infty} P_K = 1,$$

or

$$P = \left(\sum_{k=0}^{n}(1/k!)(\lambda_a/\nu)^k + \sum_{K=n+1}^{\infty}\frac{\frac{\lambda_a^k}{n!\ \nu^n}}{\prod_{m=1}^{K-n}(n\nu + m\mu)}\right)^{-1} \qquad (29)$$

These results can be used to calculate P_{nf} , or the fraction of arriving aircraft which leave without being serviced:

$$P_{nf} = R/\lambda_a ,$$

$$= \frac{\text{rate aircraft are lost}}{\text{rate aircraft arrive}} , \qquad (30)$$

where:

$$R = \sum_{k=n+1}^{\infty} (k-n)\,\mu\, P_K \quad . \qquad (31)$$

If the system is in state k where $k > n$ then waiting aircraft leave at the rate $(k-n)\mu$ and P_K is the probability the system is in state E_K . Note that this model assumes that once service begins, aircraft are <u>not</u> lost, i.e., they are assumed to remain within range of the ZRK during the firing cycle. By using equation (28), collecting terms and changing the summation variable to s=k-n, equation (31) becomes:

$$R = P \,(\alpha^{n}/n!)\,\mu \sum_{s=1}^{\infty} \frac{s\,\alpha^{s}}{\prod_{m=1}^{s} (m\beta + n)} , \qquad (32)$$

Similar manipulation in equation (29) above for P yields:

$$R = \mu \, \frac{\dfrac{\alpha^{n}}{n!} \displaystyle\sum_{s=1}^{\infty} \frac{s\,\alpha^{s}}{\prod_{m=1}^{s} (m\beta + n)}}{\displaystyle\sum_{k=0}^{n} \frac{\alpha^{k}}{k!} + \frac{\alpha^{n}}{n!} \sum_{s=1}^{\infty} \frac{\alpha^{s}}{\prod_{m=1}^{s} (m\beta + n)}} . \qquad (33)$$

Thus:

$$P = 1 - \frac{\beta}{\alpha} \frac{\dfrac{\alpha^n}{n!} \displaystyle\sum_{s=1}^{\infty} \frac{s\alpha^s}{\prod_{m=1}^{s}(n+m\beta)}}{\displaystyle\sum_{k=0}^{n} \frac{\alpha^k}{k!} + \dfrac{\alpha^n}{n!} \displaystyle\sum_{s=1}^{\infty} \frac{\alpha}{\prod_{m=1}^{s}(n+m\beta)}} .$$

which is the Venttsel' formula cited by Lozik and Petukhov.

Some sample calculations are shown in Figure A-3 where the probability that FB-111s flying at an altitute of 5 km and a speed of 2330 km/hr[10] are not engaged by an air defense system consisting of 1, 5, and 10 SA-3 missile complexes is shown as a function of λ_a. The shaded regions reflect some uncertainty in the value of the service time, t_f = 1.0 min ± 25% (where 25% is an arbitrarily chosen value intended to display the effect of uncertainty in t_f).

For example with n=5 ZRKs and λ_a= 5 FB-111s/min., P_f is 0.19 (corresponding to t_w = 1.0 min., or the best estimate for the firing cycle of an SA-3[11]). Assuming N = 24 FB-111s in the raid, P_m = 0.8 and K_{det} = 0.9 then

$$W = (1 - P_{nf}) P_m K_{det}$$

$$= 0.58,$$

$$\bar{N}_{ts} = W N$$

$$= 14.0,$$

or 10 of the 24 FB-111s penetrate the defenses. Of those that penetrate, an average of 4.6 do so without being fired upon and 5.4 are fired upon but missed.

4. "Uncontrolled" Forces

The queuing models discussed above implicitly assumed that the ZRKs were coordinated by a central command post which designated which ZRK was to fire upon which enemy aircraft. The equations used to describe the queuing process assumed that each arriving aircraft is fired upon by exactly one of the available ZRKs -- which is possible only with centralized control. If such control is not possible because, for example, the command post has been destroyed or communications disrupted, then another model must be used. Maj. O. A. Novikov addressed this problem in an article in Naval Digest in which he posed the following problem:

> The solution to many problems (repulsing air raids, cutter attacks) involves the determination of the most rational method of controlling weapons, evaluation of the effectiveness of various systems in the event centralized control is destroyed, or in the event of

autonomous, independent use of weapons.[12]

In the absence of centralized control, Novikov assumes that each arriving aircraft is fired upon by all available ZRKs. Thus the following set of differential equations describe this process:

for $0 \leq k \leq n-1$:

$$\frac{dP}{dt}_K(t) = -(\lambda_a + k\,\nu)P_K(t) + (k+1)\,\nu\,P_{K+1}(t);$$

for $k=n$:

$$\frac{dP}{dt}_n(t) = -(\lambda_a + n\nu)P_n(t) + \lambda_a \sum_{K=0}^{n-1} P_K(t) + (n\nu + \mu)P_{n+1}(t);$$

for $k>n$:

$$\frac{dP}{dt}_K(t) = -(\lambda_a + n\,\nu + (k-n)\mu)P_K(t) + \lambda\,P_{K-1}(t)$$
$$+(n\,\nu + (k-n+1)\mu)P_{K+1}(t).$$

The equilibrium solution to these equations is giver by:

for $0 \leq k \leq n$:

$$P_k = (1/k!)\prod_{m=0}^{n-1}(\alpha + m)\,P_o\;;$$

for $s \geq 1$:

$$P_{n+s} = (\alpha^s/n!) \frac{\prod_{m=0}^{n-1} (\alpha + m)}{\prod_{r=1}^{s} (n + r\beta)} P_0 ;$$

where P_0 is calculated from:

$$\sum_{K=0}^{\infty} P_K = 1.$$

These equations can then be used to calculate P_{nf}, which then gives the analog to the Venttsel' equation given above [13]:

$$P_{nf} = \frac{\beta}{\alpha} \frac{\frac{1}{n!} \prod_{m=0}^{n-1}(\alpha+m) \sum_{s=1}^{\infty} \frac{s\alpha^s}{\prod_{r=1}^{s}(n + r\beta)}}{1 + \sum_{K=1}^{n} \left((1/k!) \prod_{m=0}^{k-1}(\alpha + m)\right) + (1/n!) \prod_{m=0}^{n-1}(\alpha+m) \sum_{s=1}^{\infty} \frac{\alpha^s}{\prod_{r=1}^{s}(n+r\beta)}}.$$

Unlike the case of "controlled" forces given above, the probability that arriving aircraft are destroyed is not given by equation (7) in section 3.4 above. For "uncontrolled" forces one must account for the fact that more than one ZRK may fire upon an arriving aircraft:

$$W = \sum_{K=0}^{n-1} P_K \left(1-(1-R)^{n-k}\right) + \left(1 - P_{nf} - \sum_{K=0}^{n-1} P_K\right)R,$$

where R is the kill probability for an individual ZRK.

Novikov gives a table of values for W with "realistic consideration having been given to the possibility of the range of changes in the basic parameters: n, α, β, and R." [14]. The ranges used were: n between 2 and 5; β between 0.5 and 2; α between 0.1 and 10; and R between 0.3 and 1.0. [15].

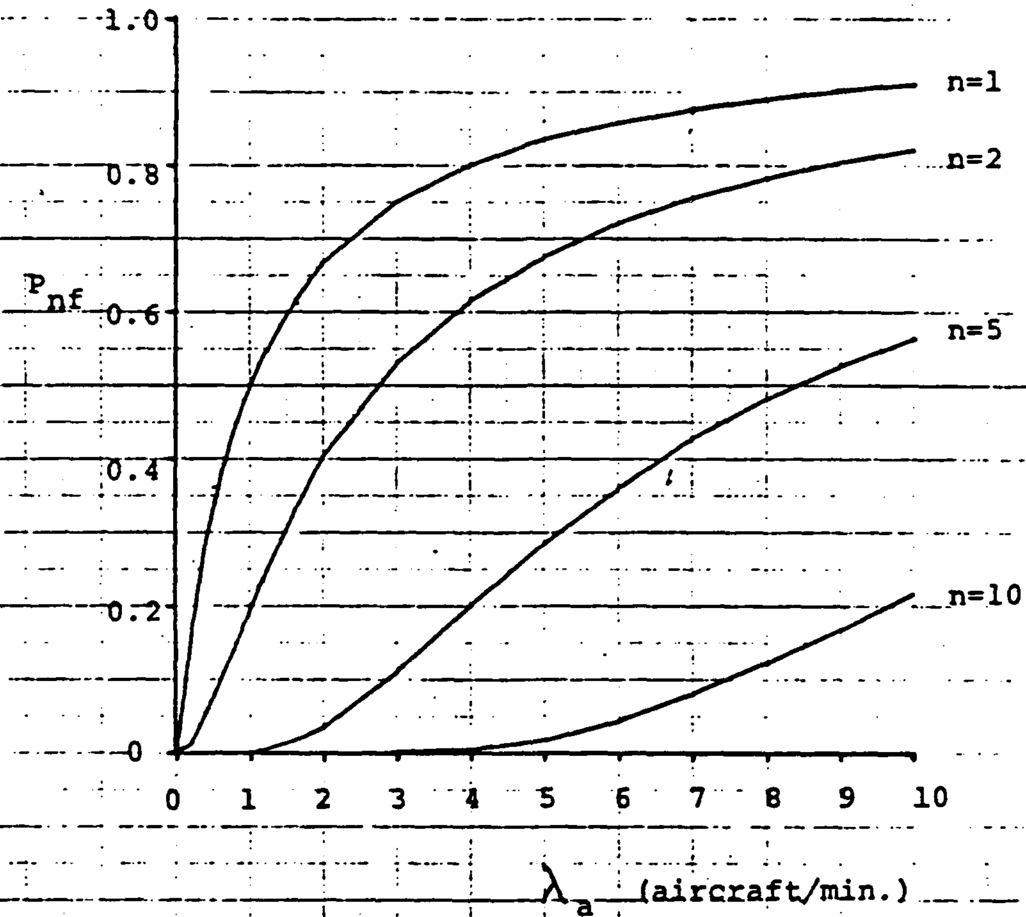

Figure A-1. P_{nf} versus λ_a for ZRKs with Small Kill Zones

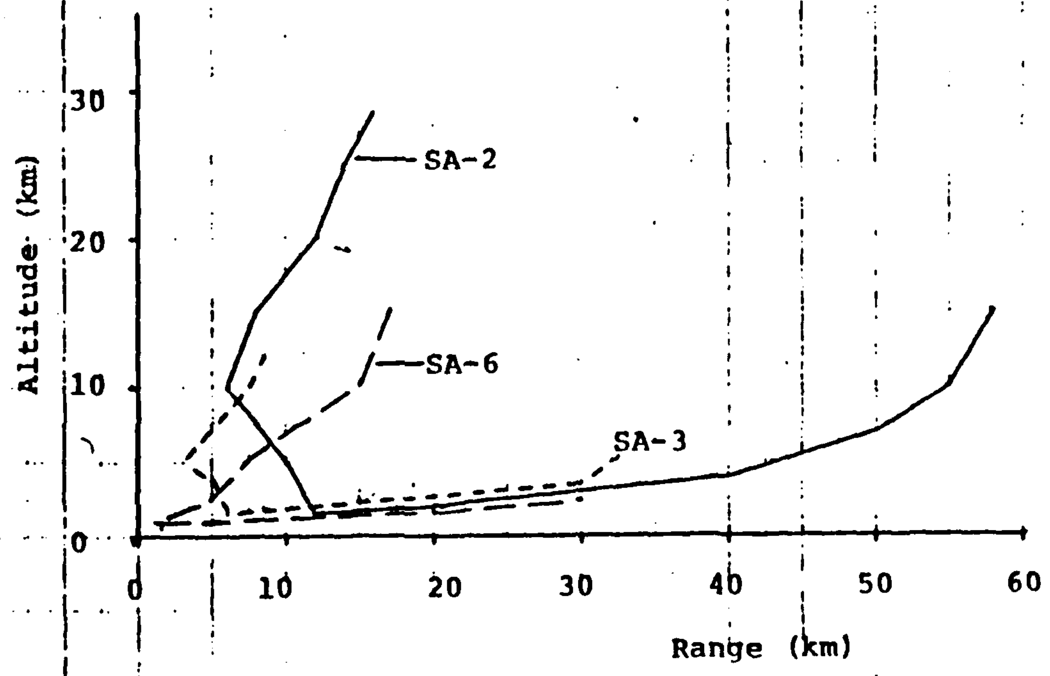

Figure A-2. Kill Zones for Soviet SAMs
Source: Isby (1981; 247-261).

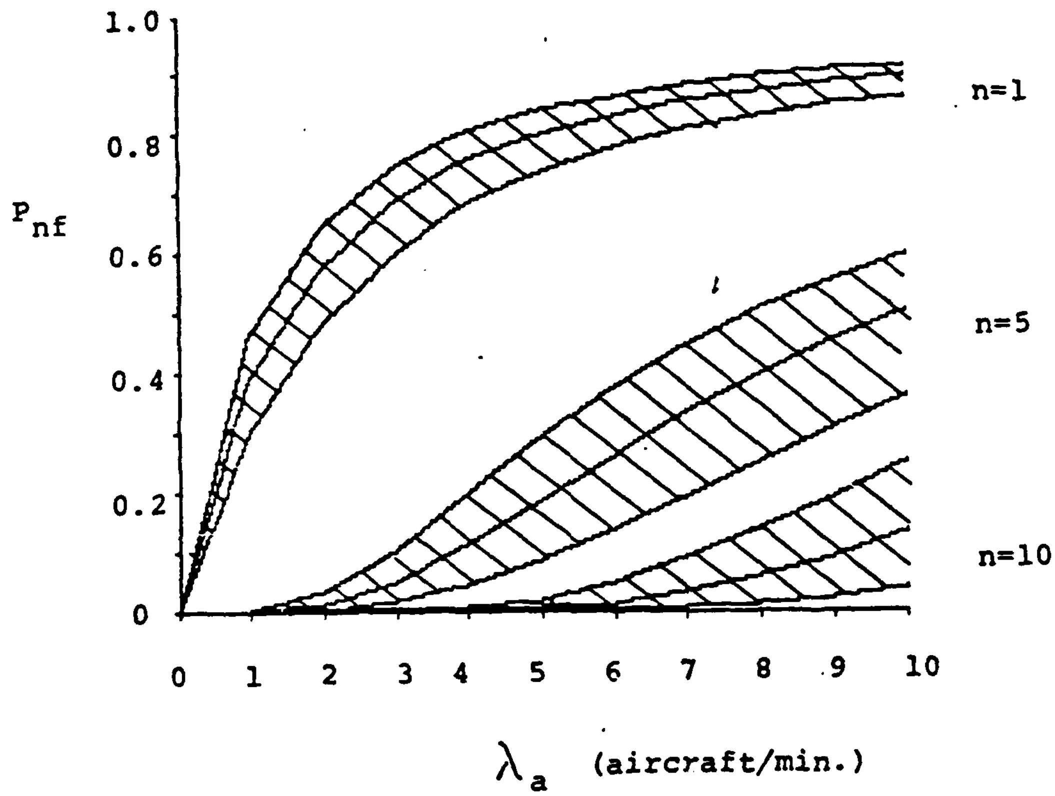

Figure A-3. P_{nf} versus λ_a for FB-111 Attack Against SA-3 Air Defense

t_w = (32-3)km/(2330 km/hr) = 0.75 min.,

where D_r=32 km and d_r=3 km for an SA-3 at an altitude of 5 km (see Figure A-2).

Footnotes

1. Chuyev and Mikhaylov (1975; 99).
2. Lozik and Petukhov (1973; 77).
3. Isby (1981; 252).
4. Isby (1981; 250-251).
5. Mal'gin (1976; 339-343).
6. Barrer (1957a; 644).
7. Gnedenko and Kovalenko (1968; 33).
8. Gnedenko and Kovalenko (1968; 39).
9. Gnedenko and Kovalenko (1968; 23).
10. Mal'gin (1976; 339-343).
11. Isby (1981; 252).
12. Novikov (1966; 27).
13. Novikov (1966; 30).
14. Novikov (1966; 32).
15. Novikov (1966; 31).